Math 246H – Exam #3 – *NB: the last page contains Laplace transforms from the book.*

(1) (15 pts) Let $Y(s)$ be the Laplace transform of $y(t)$. Express the Laplace transform of $y'''(t)$ in terms of $Y(s)$, where $y(0) = 1$, $y'(0) = 2$, $y''(0) = -1$.

(2) (15 pts) Recall that $u_c(t)$ denotes the Heaviside function

$$u_c(t) = \begin{cases} 0 & t < c \\ 1 & t \geq c \end{cases}$$

Compute the Laplace transform of

$$f(t) = (t^2 - 1)u_2(t) - (t + 2)u_3(t)$$

(3) (30 pts) Find the inverse Laplace transforms of the following functions.

(a) $F(s) = \dfrac{2s}{s^2 - s - 6}$

(b) $F(s) = \dfrac{(s-2)e^{-3s}}{s^2 - 4s + 5}$

(4) (10 pts) Re-express the following second order equation in one variable $y = y(t)$:

$$y'' + ty' + (1 - t^2)y = \cos t$$

as a system of first order equations.

(5) (15 pts) Solve the following initial value problem:

$$\mathbf{x}'(t) = \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix} \mathbf{x}(t) \quad , \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

(6) (15 pts) Find the general solution to the following system of first order equations:

$$\mathbf{x}'(t) = \begin{pmatrix} 1 & 1 \\ -4 & 1 \end{pmatrix} \mathbf{x}(t)$$