MATH 341 – EXAM # 1

Instructions. Show all your work. Be sure your name is on the booklet and that you have signed the honor pledge. You may *not* use calculators, notes, or any other form of assistance on this exam.

(1) (5 pts) Compute the Wronskian $W(y_1, y_2)$ of $y_1(t) = e^{at}$ and $y_2(t) = te^{at}$, where a is constant.

(2) (5 pts) If the rate of growth of a population $N(t)$ of bacteria is given by
$$\frac{dN(t)}{dt} = \lambda N(t)$$
for some $\lambda > 0$, how long does it take for the population to quadruple?

(3) (15 pts) Which, if any, of the following operators is linear?
  (a) $L(y) = \int_0^t (sy'(s) + s^2 y(s) + s^3) ds$
  (b) $L(y) = y'' + yy'$
  (c) $L(y) = t^2 y'' - ty' + 2y$

(4) (25 pts) Solve the following initial value problem.
$$y'' + 2y' + 5y = 0 ; \quad y(0) = 2 , \ y'(0) = 0$$

(5) (25 pts) Find the *general* solution to the differential equation
$$y'' - y' - 2y = te^{2t}$$

(6) (25 pts) Find two linearly independent solutions of
$$t^2 y'' - ty' + y = 0$$

(Hint: make a guess for one of them!)