MATH 341 – EXAM # 1 SOLUTIONS

(1) $y_1' = ae^{at}$; $y_2' = (1+at)e^{at}$. $W(y_1, y_2) = y_1y_2' - y_1'y_2 = e^{2at}$.

(2) $N(t) = N_0e^{\lambda t}$. Set $N(t) = 4N_0$. Then $t = (\ln 4)/\lambda$.

(3) Only (c) is linear.

(4) The roots of the characteristic equation are $-1 \pm 2i$, so the general solution is

$$ae^{-t}\cos(2t) + be^{-t}\sin(2t)$$

Setting $y(0) = 2$ gives $a = 2$. Setting $y'(0) = 0$ gives $-a + 2b = 0$, so $b = 1$, and the solution is

$$2e^{-t}\cos(2t) + e^{-t}\sin(2t)$$

(5) The roots of the characteristic equation are $2, -1$, so the general solution to the homogeneous equation is $y = ae^{2t} + be^{-t}$. To find a particular solution, try $\psi = (a + bt + ct^2)e^{2t}$. Note that this is quadratic because 2 is a root. Then calculate

$$\psi'' - \psi' - 2\psi = 2c + 3b + 6ct = te^{2t}$$

Hence, a particular solution is given by $c = 1/6$, $2c + 3b = 0$, or $b = -1/9$. The general solution to the inhomogeneous equation is therefore

$$y = (-t/9 + t^2/6)e^{2t} + ae^{2t} + be^{-t}$$

(6) Notice that $y = t$ is a solution! Then look for another solution of the form $y = tf(t)$. Then

$$y' = f + tf'$$
$$y'' = 2f' + tf''$$

Plugging into the equation we get

$$t^2y'' - ty' + y = tf'' + f'$$

It follows that $f' = 1/t$, or $f = \ln t$. Hence, $y = t\ln t$ is a second solution.

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