

MATH 341 – EXAM # 2

Instructions. Show all your work. Be sure your name is on the booklet and that you have signed the honor pledge. You may *not* use calculators, notes, or any other form of assistance on this exam.

(1) (10 pts) Find the inverse Laplace transform of: $\frac{s}{s^2 - 4s - 12}$

(2) (20 pts) Use Laplace transforms to solve the initial value problem

$$y'' + y = \begin{cases} \cos t & 0 \leq t \leq \pi/2 \\ 0 & \pi/2 \leq t \end{cases} \quad y(0) = 3, \quad y'(0) = -1$$

You **may** leave your answer in the form of an inverse Laplace transform.

(3) (20 pts) Write down recursive relations for the coefficients of a series solution to

$$y'' - 2ty' + \lambda y = 0$$

(4) (15 pts) Show that ± 1 are roots of the indicial equation for

$$t^2 y'' + ty' - (1+t)y = 0$$

(5) (15 pts) Express the following initial value problem as a first order system of differential equations:

$$y''' + 2y'' - y' + y = e^t \quad y(0) = y'(0) = 1, \quad y''(0) = 3$$

Include the initial value for the system.

(6) (20 pts)

(a) Compute $\exp(A)$ for the matrix $A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$.

(b) Find the solution to the initial value problem

$$\mathbf{x}'(t) = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \mathbf{x}(t) \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$