MATH 341 – FINAL EXAM

Instructions. Show all your work. Be sure your name is on the booklet and that you have signed the honor pledge. You may *not* use calculators, notes, or any other form of assistance on this exam.

(1) (20 pts) Find the general solution of the differential equation: $y'' - 4y' + 4y = te^t$.

(2) (15 pts) Solve the initial value problem

$$y^2 + 4ty^3 + 2(ty + 3t^2y^2 + 2)y' = 0 \; ; \; y(0) = -1$$

You **may** leave your answer as a functional relation between t and y.

(3) (15 pts)
   (a) Define the Laplace transform $\mathcal{L}(y)$ of a function $y(t)$.
   (b) Compute the Laplace transform of $y(t) = \cos t$.
   (c) Express $\mathcal{L}(y'')$ in terms of $\mathcal{L}(y)$, where $y(0) = 1$, $y'(0) = 2$.

(4) (20 pts) Find the general solution of $\mathbf{x}' = A\mathbf{x}$, where

$$A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$$

(5) (15 pts) Let

$$\mathbf{x}_1(t) = \begin{pmatrix} 0 \\ 0 \\ e^t \end{pmatrix} \qquad \mathbf{x}_2(t) = \begin{pmatrix} e^{2t} \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{x}_3(t) = \begin{pmatrix} te^{2t} \\ e^{2t} \\ e^t \end{pmatrix}$$

Suppose $\mathbf{x}_1(t)$, $\mathbf{x}_2(t)$, $\mathbf{x}_3(t)$ are solutions of the equation $\mathbf{x}' = A\mathbf{x}$, where A is a constant 3×3 matrix.
   (a) Show that $\{\mathbf{x}_1(t), \mathbf{x}_2(t), \mathbf{x}_3(t)\}$ are *independent* solutions.
   (b) Find the solution of $\mathbf{x}' = A\mathbf{x}$ with initial condition $\mathbf{x}(0) = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$.
   (c) Find A.

please turn ↪

Date: May 18, 2009.

(6) (15 pts) Consider the system

$$(*)\begin{cases} x' &= xy + y^2 + 2y \\ y' &= x^2 - xy + 4x \end{cases}$$

(a) Find all the equilibrium points of $(*)$.

(b) Write out the *second* order Taylor expansion of the functions

$$f(x, y) = xy + y^2 + 2y$$
$$g(x, y) = x^2 - xy + 4x$$

about the point $(-3, 1)$.

(c) Determine which equilibrium points of $(*)$ are stable.