MATH 341 – QUIZ # 1 SOLUTIONS

(1)

(a) $H(f)(0,0) = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$

(b) $f(x,y) = 1 + 2x + y + 2x^2 + 2xy + \frac{1}{2}y^2 + O((x^2+y^2)^{3/2})$

(2) Critical points are at $f_x = -2y^2 + 3x^2 - 1 = 0$ and $f_y = 4y^3 - 4xy = 0$. Solutions are $(\pm 1/\sqrt{3}, 0)$ and $(1, \pm 1)$. Using the second derivative test, the first two are saddle points, and the last two are local minima.

(3) Use Lagrange multipliers: $\nabla f = \lambda \nabla g$, where $g(x,y,z) = x^2 + y^2 + z^2$. We find

$$1 = 2\lambda x \qquad 1 = 2\lambda y \qquad -1 = 2\lambda z$$

so $x = y = -z$. This implies $x = \pm 3\sqrt{3}$. The maximum of f is $9\sqrt{3}$ and the minimum is $-9\sqrt{3}$.

Date: February 10, 2009.