MATH 341 – QUIZ # 3 SOLUTIONS

(1) (a) No. The y' term will have $(t-2)^2$ in the denominator.
(b) Yes. $\sin(t^2) = t^2 + \cdots$.
(c) Yes.

(2) Let $\phi = Ae^{i\omega t}$, and $Q =$ the real part of ϕ. Here, A is complex, so

$$\text{Re } \phi = \text{Re } A\cos(\omega t) - \text{Im } A\sin(\omega t)$$

Now $\phi' = i\omega\phi$, $\phi'' = -\omega^2\phi$, so

$$E_0 = A\left(-\omega^2 L + i\omega R + \frac{1}{C}\right)$$

$$A = \frac{E_0}{-\omega^2 L + i\omega R + 1/C}$$

$$A = \frac{E_0}{(1/C - \omega^2 L)^2 + \omega^2 R^2}\left(1/C - \omega^2 L - i\omega R\right)$$

It follows that

$$Q(t) = \frac{E_0}{(1/C - \omega^2 L)^2 + \omega^2 R^2}\left((1/C - \omega^2 L)\cos(\omega t) + \omega R\sin(\omega t)\right)$$

(3) Let $y(t) = \sum_{n=0}^{\infty} a_n t^n$. Plugging into the equation we get

$$y'' - 2ty' - 2y = \sum_{n=0}^{\infty}\left(n(n+1)a_{n+2} - 2na_n - 2a_n\right)t^n$$

or $a_{n+2} = 2a_n/n$. Since $a_1 = y'(0) = 0$, all the odd terms are zero. The recursion implies $a_{2(k+1)} = a_{2k}/k$, or in other words $a_{2k} = a_0/k!$. Since $a_0 = y(0) = 2$, the solution is

$$y(t) = 2\sum_{k=0}^{\infty}\frac{t^{2k}}{k!} = 2e^{t^2}$$

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