MATH 341 – QUIZ # 4 SOLUTIONS

(1) Denote the real part of a complex number z by $\Re z$. Then

$$\begin{aligned}
\mathcal{L}(t\cos t) &= \Re\int_0^\infty te^{it}e^{-st}dt = \Re\int_0^\infty te^{-(s-i)t}dt \\
&= \Re\int_0^\infty t\frac{-1}{(s-i)}\frac{d}{dt}e^{-(s-i)t}dt \\
&= \Re\int_0^\infty \frac{1}{(s-i)}e^{-(s-i)t}dt \\
&= \Re\left[-\frac{e^{-(s-i)t}}{(s-i)^2}\right]_0^\infty = \Re\frac{1}{(s-i)^2} = \Re\frac{(s+i)^2}{(s^2+1)^2} \\
&= \frac{s^2-1}{(s^2+1)^2}
\end{aligned}$$

(2)

$$\begin{aligned}
f*g(t) = g*f(t) &= \int_0^t se^{2(t-s)}ds = e^{2t}\int_0^t se^{-2s}ds \\
&= e^{2t}\left[-\frac{se^{-2s}}{2}-\frac{e^{-2s}}{4}\right]_0^t \\
&= \frac{1}{4}e^{2t}-\frac{t}{2}-\frac{1}{4}
\end{aligned}$$

(3) Let $Y = \mathcal{L}(y)$. Applying the initial conditions: We have $\mathcal{L}(y')(s) = sY(s) - y(0) = sY(s) - 1$, and $\mathcal{L}(y'')(s) = s^2Y(s) - sy(0) - y'(0) = s^2Y(s) - s + 1$. Then

$$(s^2-5s+4)Y - s + 6 = \mathcal{L}(y''-5y'+4y) = \mathcal{L}(e^{2t}) = \frac{1}{s-2}$$

or

$$Y = \frac{s^2-8s+13}{(s-4)(s-1)(s-2)}$$

Apply partial fractions to find

$$Y = -\frac{1}{2(s-4)} + \frac{2}{s-1} - \frac{1}{2(s-2)}$$

Taking the inverse transform, we have $y = -\frac{1}{2}e^{2t} - \frac{1}{2}e^{4t} + 2e^t$.

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