MATH 341 – QUIZ # 5 SOLUTIONS

(1) The characteristic polynomial is $p_A(\lambda) = (\lambda - 5)^2(\lambda + 10)$. Now

$$A - 5I = \begin{pmatrix} -12 & 0 & 6 \\ 0 & 0 & 0 \\ 6 & 0 & -3 \end{pmatrix} \qquad A + 10I = \begin{pmatrix} 3 & 0 & 6 \\ 0 & 15 & 0 \\ 6 & 0 & 12 \end{pmatrix}$$

It is easily seen that $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ form a basis for the eigenspace associated to $\lambda = 5$,

and $\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$ is an eigenvector with eigenvalue -10.

(2) Write $B = S + N$ where

$$S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad N = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Notice that $SN = NS$ and $N^2 = 0$. So $e^{tB} = e^{tS}e^{tN} = e^{tS}(I + tN)$.

$$e^{tB} = \begin{pmatrix} e^{-t} & te^{-t} & 0 \\ 0 & e^{-t} & 0 \\ 0 & 0 & e^{2t} \end{pmatrix}$$

(3) The characteristic polynomial is $p_A(\lambda) = \lambda^2 + 2\lambda + 2$, and so the eigenvalues are complex: $\lambda = -1 \pm i$. The vector $\binom{1}{2-i}$ is an eigenvector with eigenvalue $-1 + i$. Therefore, a basis of solutions is given by the real and imaginary parts of

$$e^t\lambda\begin{pmatrix} 1 \\ 2 - i \end{pmatrix} = e^{-t}e^{it}\begin{pmatrix} 1 \\ 2 - i \end{pmatrix} = e^{-t}\begin{pmatrix} \cos t + i \sin t \\ 2\cos t + \sin t + i(2\sin t - \cos t) \end{pmatrix}$$

$$\mathbf{x}_1(t) = e^{-t}\begin{pmatrix} \cos t \\ 2\cos t + \sin t \end{pmatrix} \qquad \mathbf{x}_2(t) = e^{-t}\begin{pmatrix} \sin t \\ 2\sin t - \cos t \end{pmatrix}$$

Notice that $\mathbf{x}_1(0) = \binom{1}{2}$, $\mathbf{x}_1(0) = \binom{0}{-1}$, so $\mathbf{x}_1 + \mathbf{x}_2$ has the correct initial conditions. Hence, the solution is

$$\mathbf{x}(t) = \mathbf{x}_1 + \mathbf{x}_2 = e^{-t}\begin{pmatrix} \cos t + \sin t \\ \cos t + 3\sin t \end{pmatrix}$$

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