MATH 436 – HOMEWORK #2 – DUE SEPT 18

(1) Let $\gamma : (-1, 1) \to \mathbb{R}^3$ be defined by

$$\gamma(t) = \left(\frac{(1+t)^{3/2}}{3}, \frac{(1-t)^{3/2}}{3}, \frac{t}{\sqrt{2}} \right)$$

Show that γ is unit speed. Compute the tangent vector $\mathbf{T}(t)$, principal normal vector $\mathbf{N}(t)$, and binormal vector $\mathbf{B}(t)$. Also compute their derivatives with respect to t.

(2) Show that for unit speed curve $\gamma : I \to \mathbb{R}^3$, the torsion is given by

$$\tau = \frac{(\dot{\gamma} \times \ddot{\gamma}) \cdot \dddot{\gamma}}{\kappa^2}$$

where $\kappa(t)$ is the curvature.

(3) Let $\gamma : I \to \mathbb{R}^2$ be a plane curve (any parametrization), with $\kappa_s(t) \neq 0$ for all $t \in I$. Define the *evolute* of γ to be the curve

$$\beta(t) = \gamma(t) + \frac{1}{\kappa_s(t)} \mathbf{N}(t)$$

(a) Show that the tangent to the evolute is normal to γ.

(b) Show the point of intersection to the normal lines to γ at t_0 and t, $t \neq t_0$, converge at $t \to t_0$ to a point on the evolute.

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