MATH 436 – HOMEWORK #3 – DUE OCT 11

(1) Using the inverse function theorem, find all points $(x, y, z) \in \mathbb{R}^3$ such that the function

$$f(x, y, z) = (x^2 + y^2, z^2 + z, xy)$$

has a differentiable inverse in a neighborhood of (x, y, z).

(2) Define a function

$$f(x, y, z) = x^{2} + y^{2} + z^{2} + xy + xz + yz - x - y - z$$

Show that the equation f(x, y, z) = 0 defines a smooth surface in \mathbb{R}^3 .

- (3) Show that the paraboloid $z = x^2 + y^2$ is diffeomorphic to a plane.
- (4) Prove that the definition of a differentiable map between surfaces does not depend on the parametrization.

Date: Sep 11, 2012.