MATH 436 – HOMEWORK #5 – DUE NOV 13

Let $S \subset \mathbb{R}^3$ be an oriented regular surface with normal ν.

(1) Compute the Christoffel symbols for the surface of revolution

$$\sigma(u, v) = (f(u)\cos v, f(u)\sin v, g(u))$$

with $(f'(u))^2 + (g'(u))^2 = 1$.

(2) Let γ be a unit speed curve on S with curvature κ. Suppose the Gaussian curvature K of S satisfies $K > 0$, and let κ_i, $i = 1, 2$, be the principal curvatures. Show that

$$\kappa \geq \min\{|\kappa_1|, |\kappa_2|\}$$

(3) Let $\gamma : (-\varepsilon, \varepsilon) \to S$ be a unit speed curve. The *geodesic torsion* of γ is defined to be

$$\tau_g = \langle D\nu(\dot{\gamma}), \nu \times \dot{\gamma} \rangle$$

Let κ_i be the principal curvatures of S in directions $\mathbf{e}_i$.

(a) Show that $\tau_g = (\kappa_1 - \kappa_2)\cos\phi \sin\phi$, where ϕ is the angle from $\mathbf{e}_1$ to $\dot{\gamma}$.

(b) If τ is the torsion of $\gamma(t)$ (as a curve in $\mathbb{R}^3$) and $\mathbf{N}$ the principal normal, let $\cos\theta = \langle \nu, \mathbf{N} \rangle$. Show that

$$\frac{d\theta}{dt} = \tau - \tau_g$$