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CORRECTION TO "SPECTRAL CONVERGENCE ON DEGENERATING SURFACES"

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Lemma 3.3 of [2] is incorrect. The asserted lower bound on the isoperimetric constant was used in the proofs of Lemma 3.4 and the claim on p. 480 to give an L^∞ bound on eigenfunctions. In this note, we shall replace Lemma 3.3 with a direct proof of these bounds (Proposition 2 below). We apologize for this error.

LEMMA 1. *Let M_ε be as in Lemma 3.3. Set $N_\varepsilon = M_\varepsilon \backslash M_1$. Then there exists a constant $c > 0$ such that for all $0 \leqslant \varepsilon < 1$, $\mathscr{I}(N_\varepsilon) \geqslant c > 0$.*

Proof. The statement follows from a simpler version of the argument used in the proof of Proposition 2.6. □

Let $\psi_{k,\varepsilon}$ be normalized eigenfunctions for the Dirichlet problem on N_ε with eigenvalues $\mu_{k,\varepsilon}$, and let $\varphi_{n_i,\varepsilon_j}$ be the normalized eigenfunctions for the Dirichlet problem on M_{ε_j} with eigenvalues $\lambda_{n_i,\varepsilon_j}$ considered in the proof of Lemma 3.4 and the claim on p. 480.

PROPOSITION 2. *For each i there is a constant B_i such that $\|\varphi_{n_i,\varepsilon_j}\|_\infty \leqslant B_i$ for all ε_j.*

Proof. For notational simplicity, we suppress the subscript n_i. Let η_1, η_2 be smooth cutoff functions on M satisfying the following conditions:

(i) $\eta_1 + \eta_2 \equiv 1$;
(ii) $\eta_1 \equiv 0$ on $M\backslash M_{1/2}$;
(iii) $\eta_2 \equiv 0$ on $M_{3/4}$.

Furthermore, we may choose η_1, η_2 in such a way as to guarantee uniform L^2 bounds on $\Delta^m(\eta_1\varphi_{\varepsilon_j})$ and $\Delta^m(\eta_2\varphi_{\varepsilon_j})$ depending on m and on λ_{ε_j}, but not otherwise on ε_j. To see this, note that by (ii), $d\eta_1$ and $d\eta_2$ are supported in $M_{1/4}$, where the higher derivatives of φ_{ε_j} may be uniformly bounded by an application of the elliptic estimate (cf. the proof of Lemma 3.5).

Clearly, it suffices to bound $\eta_1\varphi_{\varepsilon_j}$ and $\eta_2\varphi_{\varepsilon_j}$ separately. Consider the Fourier expansions

$$\eta_1\varphi_{\varepsilon_j} = \sum_{k=1}^{\infty} a_k(\varepsilon_j)\varphi_{k,1/4}, \qquad \eta_2\varphi_{\varepsilon_j} = \sum_{k=1}^{\infty} b_k(\varepsilon_j)\psi_{k,\varepsilon_j}.$$

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By Theorems 2.4 and 2.5 of the original paper and Lemma 1 above, we have the uniform bounds

$$\|\varphi_{k,1/4}\|_\infty \leqslant C\lambda_{k,1/4}, \qquad \|\psi_{k,\varepsilon_j}\|_\infty \leqslant C\mu_{k,\varepsilon_j}.$$

Here the constant C may be chosen independent of k and ε_j. Hence, it suffices to show that there is some B satisfying

$$\sum_{k=1}^{\infty} |a_k(\varepsilon_j)|\lambda_{k,1/4} \leqslant B, \tag{1}$$

$$\sum_{k=1}^{\infty} |b_k(\varepsilon_j)|\mu_{k,\varepsilon_j} \leqslant B \tag{2}$$

for all ε_j. Consider (2). By definition, we have

$$\begin{aligned} b_k(\varepsilon_j) &= \int_{N\varepsilon_j} \eta_2\varphi_{\varepsilon_j}\psi_{k,\varepsilon_j} = \frac{1}{(\mu_{k,\varepsilon_j})^m}\int_{N_{\varepsilon_j}} \eta_2\varphi_{\varepsilon_j}\Delta^m\psi_{k,\varepsilon_j} \\ &= \frac{1}{(\mu_{k,\varepsilon_j})^m}\int_{N_{\varepsilon_j}} \Delta^m(\eta_2\varphi_{\varepsilon_j})\psi_{k,\varepsilon_j}. \end{aligned}$$

Since we assume uniform L^2 bounds on $\Delta^m(\eta_2\varphi_{\varepsilon_j})$, we obtain a uniform bound

$$\mu_{k,\varepsilon_j}|b_k(\varepsilon_j)| \leqslant C(\mu_{k,\varepsilon_j})^{1-m},$$

where the constant C depends on m and on λ_{ε_j}, but not otherwise on ε_j. By the assumptions in Lemma 3.4 and the claim on p. 480, λ_{ε_j} remains bounded. By [1, Theorem 2, p. 335] and Lemma 1 above, we have $\mu_{k,\varepsilon_j} \geqslant C'k$ for some constant C' independent of k and ε_j. Hence,

$$\mu_{k,\varepsilon_j}|b_k(\varepsilon_j)| \leqslant C''k^{(1-m)},$$

where C'' may now be chosen to be independent of k and ε_j. Since the sum over k of the terms on the right-hand side converges for $m \geqslant 3$, the desired bound in (2) is obtained. The proof for (1) is similar. □

Remark. We note that a modification of this argument shows that there are uniform L^∞ bounds on eigenfunctions under conic degeneration in the separating case as well as in the nonseparating case (cf. Corollary 2.9).

References

[1] S. Y. Cheng and P. Li, *Heat kernel estimates and lower bound of eigenvalues*, Comment. Math. Helv. **56** (1981), 327–338.

[2] L. Ji and R. Wentworth, *Spectral convergence on degenerating surfaces*, Duke Math. J. **66** (1992), 469–501.

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