Homework #6
Due: Thursday, October 4, 2012

1. (2pts) Find the Fourier transform of the function $f:R\to R$

$$f(x)=\int_{-1/2}^{1/2} e^{-\pi(x-u)^2}\,du$$

2. (2pts) Let $f_0,f_1:R\to R$ be functions defined by

$$f_0(x)=e^{-x^2}\quad,\quad f_1(x)=xe^{-x^2}$$

Compute the following convolutions:

i) $f_0 * f_0$

ii) $f_0 * f_1$.

3. (3pts) Find a non-zero function f on R that satisfies the following equation:

$$\int_{-\infty}^{\infty} f(u)\,f(x-u)du = f(x)\ ,\ -\infty<x<\infty$$

4. (3pts) Find a continuous function $f:R\to R$ that satisfies

$$-f'(x)+f(x)=\begin{cases} e^{-x} & ,\ x>0 \\ 0 & ,\ x<0 \end{cases}$$

Total: 10pts.