Homework #7
Due: Thursday, March 17, 2011

I. [3pts] Assume $f:R \to R$ is a square-integrable function whose Fourier transform is supported in the frequency band *[2 KHz,12 KHz]* (this means its Fourier transform *F(w)* vanishes for any frequency *w* outside the interval [2000,12000]). Assume we know the samples *{f(nT) , n=…,-2,-1,0,1,2,… }* for $T=100\mu s=10^{-4}$s. Show how to synthesize f(x) from this set of samples.

II. [1 pt] What is the maximum sampling period of a 50 *KHz* band-limited signal so that we can perfectly compute the signal at any time *t* from its entire cardinal series? (Recall 1 *KHz* = 1,000 *Hz* = 1,000 s^{-1}).

III. [4pts] A 10*KHz* band-limited signal is sampled at its Nyquist rate. The only nonzero samples are:

$$x(t) = \begin{cases} -2 & t = -1.2\ ms \\ 1 & t = 0.2\ ms \end{cases}$$

Compute x(t) for t=0 *ms* and t=1 μs. Recall 1 *ms* = 10^{-3} *s* and 1 μs=10^{-6} *s*.

IV. [2pts] An unknown signal $f:R \to R$ is sampled at the sampling frequency 1KHz. We do not know if it is band-limited however we know its Fourier transform is upper bounded by

$$|F(w)| \le e^{-|w|} \text{ for all w.}$$

Estimate what is the maximal reconstruction error when using the Shannon's formula with all samples *{f(n/1000) , n=…,-2,-1,0,1,2,… }*

Total: 10 pts