

## Mid-Term Exam

October 25, 2007 , 9.30am-10.45am

You are allowed one one-sided cheat sheet with formulae/text of your choice. No discussions are allowed.

Problem 1. (10pts)

Show that if  $f: \mathbf{R}^d \rightarrow \mathbf{R}^d$  is locally Lipschitz on  $\mathbf{R}^d$  and satisfies  $\|f(x)\| \leq \|x\|^2$ , then the domain of the general solution  $t \rightarrow x(t, \xi)$  for  $\dot{x} = f(x)$  includes the interval  $0 \leq t < \|\xi\|^{-1}$ .

Problem 2. (30pts)

Consider the undamped pendulum  $\ddot{x} + k^2 \sin(x) = 0$ , with  $x \in \mathbf{R}$ .

1. Rewrite it as a first-order system, and find an integral.
2. Sketch its phase portrait (of the first-order system written above).
3. For what values of  $x_0$  (initial value for  $x$ ), if any, is the solution of  $\ddot{x} + k^2 \sin(x) = 0$  with initial velocity 0 periodic? (Here 'velocity' means  $\dot{x}$ )

Total: 40 points