# HW4 Project Problem A for Math 420

In class we discussed the prey-predator population model with harvesting of the form

$$\frac{dp}{dt} = (r - ap - bq)p - h\,, \qquad p(0) = p^{\text{init}}\,,$$
$$\frac{dq}{dt} = -(s - cp)q\,, \qquad q(0) = q^{\text{init}}\,.$$

Here $p(t)$ is the total population size of all the fish species that we wish to harvest, while $q(t)$ is the total population size of the predator fish species, which we do not wish to harvest. This model oversimplifies by lumping all prey and all predator species into just two groups, and by assuming we do not want to harvest predator fish too, which in reality we do.

Solutions of this system depend upon the initial values $p^{\text{init}}$ and $q^{\text{init}}$, and upon the six nonnegative constant parameters $r$, $s$, $a$, $b$, $c$, and $h$. Notice that when $q^{\text{init}} = 0$ then $q(t) = 0$ for all time, and the model reduces to the logistic model with harvesting from project 3A. Also notice that if $s = c = 0$ then $q(t)$ is constant and again $p(t)$ satisfies the logistic model with harvesting. This project explores how the parameters are determined from data.

(1) Give interpretations for each of the six parameters in the model. Find a condition on them that insures the model has a stationary solution $(p_o, q_o)$ with both $p_o$ and $q_o$ positive. Determine the stability of this solution as a function of the parameters.

(2) Give some phase-plane portraits of this system (restricted to the quadrant $p \geq 0$, $q \geq 0$) that represent how this system behaves for different values of the parameters. (Use software tools to do this!) Identify all stationary solutions and their stability. You will want to look at values of the parameters where the stability of any stationary solution changes. You should also look at $h = 0$. Explain why some of the phenomena that you see cannot be captured by a simple logistic model with harvesting. Identify conditions on the parameters under which the fishery collapses.

(3) Suppose that the positive stationary solution $(p_o, q_o)$ is attracting. Suppose that the solution is very near this stationary solution when all of a sudden (say at $t = 0$) all fishing is stopped, thereby changing the system by setting $h = 0$. Assuming the other five parameters are unchanged, show how to use the observed response $(p(t), q(t))$ of the new system over a time interval $[0, T]$ to find as many of these five parameters as you can. For this part of the project, assume your observations are perfect.

(4) Repeat the previous part assuming you only know $(p(t), q(t))$ at a finite number of times $\{t_k\}_{k=0}^{n}$ where $t_k = kT/n$. Try out your algorithm by selecting values of the six parameters for which the positive stationary solution $(p_o, q_o)$ is attracting, but near where the system undergoes some change. Then set $h = 0$ and compute $\{(p(t_k), q(t_k))\}$ as data for the orbit with $(p^{\text{init}}, q^{\text{init}}) = (p_o, q_o)$ and try to recover the five other parameter values from the data.

(5) Explore how sensitive the calibration in the previous part is to uncertainties in your measurement of $\{(p(t_k), q(t_k))\}$. Add random errors to the data $\{(p(t_k), q(t_k))\}_{k=0}^{n}$ and see how much your calibration of the parameters changes. Initially take these errors to be independent with a common distribution. Then, if you like, imagine more complicated error patterns. How might these uncertainties affect your ability to manage the fishery?