**Stat 400, Fall 2003**                                                    10/3/03

**In-Class Test 1**

**Instructions:**    You may use calculators and 1 notebook sheet of notes. Give reduced numerical answers only in problem 4(a); but otherwise reduce your answers to simple formulas or numerical expressions (summations and binomial coefficients are allowed). Each problem counts 22 points.

**1.**   Suppose that you are dealt a hand of  4  cards from a well-shuffled ordinary deck of  52  playing cards. What is the probability that your hand contains exactly  2  picture cards (J, Q, or K) and  2  non-picture cards ? What is the probability if the 4 cards had been picked at random *with replacement* from the deck ? For each part, say what your sample space is.

**2.**   Suppose that an electrical device has three components, A, B, and C, which will fail, independently of each other, with respective probabilities  $0.3, 0.2,$  and  $0.4.$  Suppose the device works properly only if either component A or **both** of B and C work.

   (a)  What is the probability that at least one of the components works ?

   (b)  What is the probability that the device works properly ?

**3.**   I have three coins in my pocket, one fair (with heads-probability  $\frac{1}{2}$) and two biased, with heads-probability $\frac{2}{3}$.  I pick one of the coins at random and toss it independently 3 times.

   (a)  What is the probability that my tosses result in 2 Heads and 1 Tail (any order) ?

   (b)  *Given* that my tosses resulted in 2 Heads, 1 Tail, what is the probability that the coin I picked was the fair one ?

**4.**   In 512 classrooms nationwide, the following experiment is performed independently: each of 10 students independently flips a fair coin.

   (a) What is the probability, in each classroom, of getting at most 1 head out of the 10 tosses ?

   (b) What is the probability that in at least 10 of the classrooms nationwide, the number of heads recorded was  $\leq 1$  ?

**5.**   Suppose you play a gambling game in which you win  $1200  with probability 0.2,  win  $200  with probability 0.3,  and lose  $600  with probability 0.5.  You must pay income tax of  25%  of your winnings if you win money (but pay no tax and get noE tax deduction if you lose). Find (a)  your expected dollar winnings (before taxes) from playing, and  (b)  the probability mass function of your after-tax winnings $X$ (where losses are recorded as negative winnings).