Stat 400 In-Class Test 2 Fall, 2003

Instructions: Do as much of the test as you can; point values are indicated, with 100 the maximum score. You may use calculators, a notebook sheet of notes, and the normal table handed out with the test. Give reduced numerical answers only in problems 1(a), 3(b), and 4(a): elsewhere, numerical expressions which are readily evaluated on a calculator are good enough. (But do not leave the answer in the form of an integral or abstract symbol). Justify your steps wherever you can: I will be much more generous with partial credit wherever you explain clearly what you are doing.

1. (*25 pts.*) Suppose that a certain continuous random variable X, has density proportional to $1/(1+x)^3$ for $x \ge 0$ (and is 0 for negative x.)

(a) Find the probability that X is either < 1 or > 3 (a single number). (*Give reduced numerical answer.*)

(b) Find the probability density of the random variable $(1+X)^2$.

2. (*25 pts.*) Each of 1000 real numbers is rounded to the nearest integer. If the round-off errors V_i, $1 \le i \le 1000$, can be treated as independent Unif$[-0.5, 0.5]$ r.v.'s, then what is the approximate probability that the total of the rounded numbers differs from the total of the original (un-rounded) numbers by at most 20 ?

3. (*30 pts.*) After generating 4800 independent Unif$[0,1]$ random numbers $U_1, \ldots, U_{4800}$, we count the number M of them which have values $\le 1/4$, i.e.,

$$M = \sum_{i=1}^{4800} I_{[U_i \le 0.25]}$$

Here the notation I_A denotes 1 if A is true, and 0 if A is false.

(a) What type of random variable is M ?

(b) What is the approximate *numerical* probability that $M \le 1280$?

(c) What is the approximate value of $\frac{1}{4800} \sum_{i=1}^{4800} U_i^4$?

4. (*25 pts.*) Suppose that two discrete random variables X, Y have joint probability mass function given by the entries in the table

Y=	-3	-1	0	4
X= -1	0.06	0.05	0.09	0.10
X= 1	0.09	0.08	0.12	0.11
X= 2	0.05	0.07	0.09	0.09

(a) Find the probability that $X > Y$.

(b) Find the marginal probability mass function of X and the expectation of Y .

(c) Are X, Y independent ? *Justify your answer.*

5. (*15 pts.*) Let V, W be independent $\mathcal{N}(2,5)$ random variables. Find $E((V+2W)^2)$.

Hint: one way to find this is to relate it to the mean and variance of the random variable $V + 2W$.