Sec. 0301 12/8/03

Stat 400, Test 3 Solutions

#1. (a) Upper endpoint $\overline{X} + z_{.1} s/\sqrt{n} = .75 + 1.282(.12)/15 = 0.760$,

(b) $\overline{X} \pm z_{.08/2} s/\sqrt{n}$, with $z_{.04} = 1.751$ because from the normal table, $\Phi(1.75) = 0.9599$, $\Phi(1.76) = 0.9608$. So CI$= (0.736, 0.764)$.

#2. The number T out of 1000 favoring Dean is (approximately, if sampling were done with replacement) $Binom(1000, .35)$. So answer is:

$$1 - P(|\frac{T}{1000} - .35| < .02) = 1 - \Big(2\Phi\Big(\frac{.02}{\sqrt{.35(.65)/1000}}\Big) - 1\Big) = 2(1 - \Phi(1.326)) = .185$$

#3. (a) For the histogram, each count of 1 contributes a unit of $h = 1/(20 \cdot 6) = 0.00833$ to the height of the histogram bars. The five class-intervals have the following counts and bar-heights:

Interval	48.5-54.5	54.5-60.5	60.5-66.5	66.5-72.5	72.5-78.5
Count	1	4	6	5	4
Bar-height	.00833	.0333	.05	.04167	.0333

(b) Median $= .6495$, and $2/3$ quantile is 67.8.

#4. Here (a) and (b) could be done using what you know about one-sided z- and t- confidence-intervals, **or** from first principles, as follows.

(a) We know that $\mu \in (-\infty, \overline{Y} + 1.83 s/\sqrt{10}$ is a one-sided 0.95 level confidence interval because $t_{9,.05} = 1.83$, and $\mu \in (\overline{Y} - 1.38 s/\sqrt{10}, \infty)$ is a one-sided 0.90 level confidence interval because $t_{9,.10} = 1.38$. Thus the probability that one or the other of these two interval conditions is violated is $.05 + .1 = .15$ and the desired probability is $1 - .15 = 0.85$. This could also be done by recognizing the requested probability as

$$P(-1.83 \le (\overline{Y} - \mu)/(s/\sqrt{n}) \le 1.38) = P(t_9 \le 1.38) - P(t_9 \le -1.83) = .9 - .05$$

(b) This is just like the previous part, except that we must find the probabilities replacing 0.05, 0.1 for the normal instead of t_9 distribution. By the simpler 'method 2', the requested probability is

$$P(-1.83 \le (\overline{Y} - \mu)/(\sigma/\sqrt{n}) \le 1.38) = \Phi(1.38) - \Phi(-1.83) = .9162 - .0336 = 0.8826$$

(c) Prediction interval: $\overline{Y} \pm t_{9,.005}\, s\, \sqrt{1 + \frac{1}{n}} = 3.2 \pm 3.250\, 0.9/\sqrt{1.1} = (.13, 6.27)$.