Solutions to HW4, Stat 401 Spring 2011

(1). #10, pp. 293-4. (a). $H_0 : \mu = 1300, \quad H_A : \mu > 1300.$ (b). Under $H_0, \quad \bar{X} \sim \mathcal{N}(1300,\, 60^2/20),$ and α type I error probability $=$ $1 - \Phi((1331.26 - 1300)/(60/\sqrt{20}) = .010.$ (c). When $\mu = 1350, \quad ll\bar{X} \sim \mathcal{N}(1350,\, 60^2/20),$ and the probability of (wrongly) accepting the null is $\Phi((1331.26 - 1350)/(60/\sqrt{20}) = .081.$ (d). The one-sided test analogous to (b) with $\alpha = .05$ is: reject when $\bar{X} \geq 1300 + z_{.05}60/\sqrt{20} = 1322.07$. Using this test, our type II error probability in (c) would be $\beta = \Phi((1322.07 - 1350)/(60/\sqrt{20}) = .019.$ (e). The rejection region in (b) corresponds to $Z \geq (1331.26 - 1300)/13.42 = 2.33 \; (= z_{.01})$.

(2). #20, p. 304. One-sided hypothesis, rejecting if $\bar{X}$ is too small. The `Minitab` output already shows p-value .016, so we would reject for $\alpha = .01$ and accept for $\alpha = .05$. Yet either α value could be used, depending on the relative costs of wrong decisions.

(3). #32, p. 306. Here $n = 12$, $\bar{X} = 98.38$, $s = 6.11$. (a). Two-sided test statistic for $H_0 : \mu = 100$ is $|98.38 - 100|/(6.11/\sqrt{12}) = .918,$ which is clearly less than $t_{11,.025} = 2.20,$ so we accept.

(b). Sample size needed: $(7.5)^2(2.33 + 1.96)^2/5^2 = 41.4$ rounded up to $n = 42$.

(4). #42, p. 311. Binomial hypothesis testing for $H_0 : \; p = .5$ versus $p > .5$. (a). Rejection region $\{x : \; x \geq 15\}$ is the only one of the three that makes sense here. (b). `1-pbinom(14,20,.5)` $= .021$ is the significance level of the region in (a). Yes, it is a level .05 test, with α as close to .05 as it can be without exceeding .05. (c). Here we want the type II error probability for $p = .6$, $\beta =$ `pbinom`(13,20,.6) $= 0.75$, very high. With $p = .8$, $\beta =$`pbinom(13,20,.8)` $= .087$, not too bad. (d). For level .10, rejection region becomes $\{X : \; X \geq 13\}$ which has exact significance level .058, which is aclose as we can get to .10 without exceeding it. So yes, we would reject if 13 out of 20 prefer gut strings.

(5). #52, p. 317. Because each trial in the experiment provided the taster with 3 choices, there is probability $p_0 = 1/3$ if the taster has no real ability to distinguish wines. So that is $H_0 : \; p = 1/3,$ which we test against the alternative $H_A : \; p > 1/3$. The observed value $X \sim \text{Binom}(855, p)$ is

$X = 346$, yielding $\hat{p} = 346/855 = .4047$. For this one-sided large-sample test, the standardized test statistic is $(.4047 - .333)/\sqrt{(.333)(.667)/855} = 4.449$. Since the p-value is $1 - \Phi(4.45) = 4.3 \cdot 10^{-6}$, the experiment provides convincing evidence that the tasters' abilities are much better than chance, although we might feel that a skilled taster ought to be able to perform with an even larger probability, sat $p > 0.5$.

(6). #54, pp. 317-8. (a). For a one-sided test of $H_0 : p \le .2$ versus $H_A : p > .2$, with $X = 15$ occurrences out of $n = 60$ trials, the continuity-corrected test statistic is $((15.5 - 12)/60)/\sqrt{.2(.8)/60} = 1.13$ leading to P-value $1 - \Phi(1.13) = 0.13$, and the company should accept the 20% figure. (Continuity correction is optional.)

(b). This is a question about power, figured at $p = .5$.

$$P_{p=.5}(X > n(.2+z_{.01}\sqrt{.16/n} = P((X-n/2)/\sqrt{n/4} > 2\sqrt{n}(-.3+2.33(.4)/\sqrt{n})$$

$$\approx 1 - \Phi(-.6\sqrt{n} + 1.86)$$

which for $n = 60$ gives a power of .997.

(7). #2, p. 334, three ways. (a). Large-sample method: Z test statistic

$$= |42500 - 40400|/((2200)^2/45 + .5(1900)^2/45))^{1/2} = 4.846$$

rejects because $> z_{.025} = 1.96$.

(b). Pooled-variance t-statistic

$$= |42500 - 40400|/((.5(2200)^2 + .5(1900)^2) * (2/45))^{1/2} = 4.846$$

(algebraically same value !) rejects because $> t_{88,.025} = 1.987$.

(c). Satterthwaite approximate t-statistic is calculated to have exactly the same value 4.846 as in large-sample method, and should be compared with $t_{\nu,.025}$ where $\nu = (2200^2/45+1900^2/45)^2/((2200^2/45)^2/44+(1900^2/45)^2/44) = 86$ (after rounding down). So the percentage point is $t_{86,.025} = 1.988$, and this approximation makes hardly any difference in this example.

(8). #28, p. 342 Here the sample sizes are small, $m = 10$ and $n = 5$, so we must assume that the individual data values (lean angles) are normally distributed, with the same parameters within each sample. The summary statistics are $\bar{x} = 30.7$, $s_1 = 2.75$, $\bar{y} = 16.2$, $s_2 = 4.438$, and the calculated value of ν is 5 (after rounding down). Here, because we are asked to test $H_0 : \mu_1 - \mu_2 \leq 10$ versus $H_A : \mu_1 - \mu_2 > 10$, we can do an ordinary (Satterthwaite) two-sample t test by reducing $\bar{x}$ by 10, giving the one-sided test statistic $(30.7 - 16.2 - 10)/\sqrt{2.75^2/10 + 4.438^2/5} = 2.077$ which would reject at .10 level because it exceeds 1.476. Indeed, its t_5 p-value is $1 - \texttt{pt}(2.077, 5) = .046$. (Note that we use one-sided test, one-sided p-value.)