Oct. 20, 2010

Additional Sample Problems for Test 1, Stat 401

These problems have more parts than those I would ask on a test, but the topics are fair game.

(1). Suppose that we generate independent $Uniform[0,1]$ random variables $U_{i,j}$ for $1 \le i \le 250,\ 1 \le j \le 40$ and store them in a 250×40 matrix. Let the row-totals be defined as

$$T_i \;=\; U_{i,1} + U_{i,2} + \cdots + U_{i,40}$$

Since all of the random variables $U_{i,j}$ are independent and identically distributed, and the row-total random variables T_i are independent of one another and all have the same distribution as i ranges from $1 \ldots 250$.

(a) What is the (approximate) probability density of T_1 ? Would you expect the approximation to be good ?

(b) What is the expected number of the row-totals $T_i,\ i = 1, \ldots, 250$ observations which would fall **outside** the range from 22 to 31 ? What is the approximate probability that **none** of the 250 row-sums fall outside this range ?

(c) Define N to be the number of the rows $\{U_{i,j}\}_{j=1}^{40}$ among $i = 1, \ldots, 250$ for which

$$\frac{1}{2} \;>\; \frac{T_i}{40} \;+\; z_{.01}\,\frac{1}{\sqrt{480}}$$

What is the approximate probability distribution of N ?

(d) Let T_i be as above, and define the row sample standard deviation S_i as the square root of

$$S_i^2 \;=\; \frac{1}{39}\sum_{j=1}^{40}(U_{ij} - T_i/40)^2$$

Define M to be the number of the rows $\{U_{i,j}\}_{j=1}^{40}$ among $i = 1, \ldots, 250$ for which

$$|\frac{1}{2} \;-\; T_i/40| \;>\; z_{.01}\,\frac{S_i}{\sqrt{40}}$$

What is the approximate probability distribution of M ? What is the approximate probability that $M \leq 1$?

(2). Suppose that two independent observed samples of manufactured items, an X-sample of 200 from one factory and a Y-sample of 300 from a second fectory, are tested for defects, result in 18 defectives from the first sample and 45 from the second.

(a). Formulate appropriate null and alternative hypotheses to test whether the two factories produce defective items at the same rates (ie with the same probabilities of each item being defective). Test your hypothesis and give the p-value. Would you be able to reject the null at all significance levels between 0.05 and 0.10 ?

(b). Give the rejection region, in terms of the number N_1 of defectives in the first sample and N_2 in the second sample, for a test of significance level approximately 0.05 for your hypothesis in (a). What was the type II error probability of this test if the actual probabilities of defective items were 0.10 from factory 1 and 0.20 from factory 2 ?

(3). (a) Give a 90form $[L, \infty)$ for the unknown mean $\mu = 1/\lambda$ of an Exponential random variable with density $f(x) = \lambda e^{-\lambda x}$, $x > 0$, based on a sample of size 36 from this density.

(b). How large a Binomial sample size n is needed so that the width of a 2-sided 90% confidence interval for the unknown probability p of success will be no more than 0.01 :

(a) if you know that the value of p is close to 0.3 ?

(b) if you have no idea, before an experiment is conducted, of the correct value of p ?

(4). If you are conducting a 2-sided hypothesis test of $H_0 : \mu = 10$ at significance level $\alpha = .05$ for the mean μ of a sample of 20 normal $\mathcal{N}(\mu, 4)$ random variables, based on $\bar{X}$, then find the power of the usual test against the alternative $\mu = 13$.