May 14, 2006 evening

Stat 470 Take-Home Exam Problems

Instructions. Do all 12 problems, to be handed in by 3:30pm, Wed., May 17, 2006. All problems count equally. Give clear numerical answers and explanations, justifying your method wherever possible. On this exam, there is to be **no** sharing of ideas or hints with fellow students or anyone else except me: but you may ask me for hints individually, either via email or in person.

For problems (1)–(4), assume that

$$l_{20} = 850,000 \quad , \qquad d_{20} = 2000 \quad , \qquad d_y = 1.003^{y-20} d_{20} \quad \text{for} \quad 20 \le y \le 45$$

Also, assume wherever needed in (1)–(3) that the survival function is linear between successive birthdays, i.e., within each year of age.

(1). Find the expected number of whole years of life lived between exact ages 20 and 45, for a life aged 20.

(2). Find the probability that a life aged 20 dies between exact ages 30 and 36, and find the force of mortality at age 37.5 .

(3). Find the net single risk premium for a life aged 22 of a contract which pays $200,000 at the end of the quarter-year of death if death occurs within 10 years and $100,000 at age 45 if the life aged 22 lives to age 45. Assume that the interest rate for the contract is 4%.

(4). Suppose that $10,000 borrowed at time 0 is to be re-paid over 10 years in quarterly payments of s_j at $j/4$, $j = 1, 2, \ldots, 40$, and that the effective interest rate is i over the entire 10-year period. Explain in detail, either algebraically or with clear definitions and intuitive reasoning, why the total interest paid is $\sum_{j=1}^{40} s_j - 10,000$. Use this fact to give an explicit formula depending on nothing but i for the total interest paid in a level-repayment plan (10-year mortgage), with all payments $s_j = s$ equal.

(5). A life aged 25 purchases a 25-year endowment life insurance for $100,000 (payable at the end of the year of death if death occurs within 25 years, and at the end of 25 years if the insured is alive), paying for it with a level annual premium P. Assume that the premiums are calculated with interest rate 5% and a 3% loading (including both profit and administrative costs). To pay for the policy, the insured has borrowed from a bank an amount $\$X$ in the form of a 25-year annuity-due certain, with yearly payments P, calculated with interest rate 6%. (Assume X is the exact present value at 6% of the annuity-due, without additional expenses or profit.) Assume also that, at the termination of the endowment insurance contract, the insured or estate uses the endowment-insurance proceeds immediately to terminate the annuity-certain and pay off the amount received so far from the bank, calculating the payoff amount with the 6% interest rate. Find an expression in actuarial notations for the expected present value (at time 0, when the insured is 25) of the net profit to the insured (or his estate) at the termination of these contracts. **Note: I did not previously indicate explicitly what interest rate to use for the Present Value: I had intended 5%, but you may also use 6%. In any case your actuarial notations should show explicitly what interest rate they incorporate.**

For the next two problems, assume that a life-table **(L)** has ${}_kp_{40} = exp(-.025\,k)$ and ${}_kp_{60} = exp(-.05k)$ for integers k ranging $0 \le k \le 20$.

(6). If a special population **(*)** has force of mortality $\mu^*(t)$ which at all ages is 1.5 times as large as that governing life-table **(L)**, then find (a) the probability of a (*) population life aged 45 survivng to age 65, and (b) the ratio of the net single risk premium of a 15-year term insurance (payable at end of year of death, with i=.06) for a population (*) life aged 45, to the corresponding net single risk premium for a population (L) life aged 45.

(7). Calculate the reserve ${}_{10}V^1_{45:20}$ for a population (L) life, based on interest rate $i = .06$.

(8). A homeowner has taken out a 30-year mortgage for $200,000 at 5%, with monthly payments, but after making all payments over the first 12 years of the mortgage, missed payments in months 7 and 9 of the 13th year of the mortgage, and then resumed paying faithfully. How much interest did the homeowner pay on his loan in the 14th year of the mortgage ? *Assume that the payments paid after the missed payments were exactly the same as the earlier payments before the missed ones; however, the part of each payment that counts as interest always depends on the balance owed. The missed payments' principal would be paid at the end of the mortgage term, as a lump sum. That is, the final payment after 30 years would be equal to the total unpaid balance at that time.*

(9). Over the age-range from 20 to 45, members of a certain population have failures which are uniformly distributed, i.e. a survival function which decreases linearly on the interval $[20, 45]$. Assume that ${}_{25}q_{20} = .15$. A 24-year-old member of this population wants to buy a 20-year term insurance which pays $\$400000/t$ at the instant of death, if death occurs at exact age t between ages 24 and 44 (and pays 0 otherwise). If the loading on this insurance policy is 2% and the interest rate is 5% and premiums are to be paid twice-yearly, find the amount of each premium payment.

(10). If the force of mortality happens to be constant, $\mu(t) = \mu = 0.03$, over the age-range $[30, 50]$, and if the whole-year survival probabilities ${}_kp_{30} = exp(-.03\,k)$ for integers k between 0 and 20 are taken as known from a life-table, then find the actual and Balducci-interpolated probabilities of a 32-year old dying between exact ages 33.7 and 35.7.

(11). Find the effective interest rate up to time 10 for an investment in which you deposit amounts $10 + 20k$ at integer times $k = 0, 1, 2, \ldots, 10$ into a fund which earns interest at a time-varying instantaneous force of interest

$$r(t) \quad = \quad .08 \; - \; .002\, t \quad , \qquad 0 \le t \le 10$$

Hint: find the constant rate i which yields exactly the same accumulated balance up to time $t = 10$ as the investment with specified time-varying interest rate.

(12). Suppose that the remaining lifetimes at ages 40 and greater in a special population are distributed according to a Weibull distribution, with ${}_tp_{40} \; = \; exp(-8.557\, t^4 \cdot 10^{-7})$. Find the median and expected (exact) age at death for this population.