

## In-Class Test, Stat 470

Do the following 4 problems, using only a (1-or-2-sided) sheet of formulas and a calculator as aids. 100 points is a perfect score. You need not give a reduced numerical answer, but, except in Problem (2), your answer should be a numerical expression (without integrals or sums) from which it would be easy to evaluate the numerical answer using a scientific calculator.

**(1).** (30 pts.) Suppose that you borrow (at  $t = 0$ ) \$100,000 for 20 years at 6% interest (APR) to be repaid in equal monthly payments ( $m=12$ ), with first payment due in 1 month. Find

(a) the present value at  $t=0$  of the first 36 payments (i.e., all payments over the first 3 years), and

(b) the portion of the 37<sup>th</sup> payment (the one to be paid in exactly 3 years and one month) which is interest.

**(2).** (30 pts.) The survival function for a certain population is given by  $S(t) = (1 + t/80)^{-5}$  for  $t \geq 0$ .

(a) Find the force of mortality  $\mu(t)$  as a function of  $t$ .

(b) Suppose that an insurer will make a payment equal to  $10,000\sqrt{T}$  on a newborn life which dies at exact age  $T$ . Give (but do not evaluate) an explicit integral expression for the expected value (= expectation, disregarding interest-rate discounting) of the payment.

**(3).** (25 pts.) An investment of \$100 at time  $t = 0$  earns (and re-invests) interest at the nominal quarterly rate  $i^{(4)} = 0.06$  for 2 years and with the variable force of interest  $\delta(t) = 0.01 + t/50$  between times  $t = 2$  and  $t = 5$ . Find the effective (constant, APR) rate of interest which the investment earns over the 5-year period ( $t = 0$  to  $t = 5$ ).

**(4).** (25 pts.) A cohort life-table yields ratios

$${}_{10}p_{20} = \frac{l_{30}}{l_{20}} = 0.9 \quad , \quad {}_{30}q_{30} = 0.25 \quad , \quad {}_{50}p_{20} = 0.5$$

(a) Find the probability for a life aged 30 to die between 60 and 70; and

(b) Assuming that  ${}_t p_{30}$  is linear in  $t$  for  $0 \leq t \leq 30$ , find  ${}_{30}p_{20}$ .