## Sample Short-Answer Problems on Poisson Process, STAT 650

**Problem 1.** Suppose that $N_j(t)$ are independent Poisson processes for $j = 1, 2,$ with respective rates 1, 4.

(a) Find the probability that 6 of the first 8 jumps in the superposed process $N(t) = N_1(t) + N_2(t)$ are jumps in the $N_2(t)$ process.

(b) Find the probability that 2 jumps occur in the $N_2$ process occur before the first jump in the $N_1$ process.

(c). Find the expected total waiting time until the first jump in $N_1(t)$ following the 3rd jump in $N_2(t)$.

**Problem 2.** Let $N(t)$ be a Poisson(1) process and $X_i,\ i = 1, 2, \ldots$ be a sequence of independent discrete random variables independent of $N(t)$, taking respective values $0, 1, 2$ with probabilities $0.2, 0.5,$ 0.3. Find the probability that $\sum_{i=1}^{N(5)} I_{[X_i=1]} = \sum_{j=1}^{N(5)} I_{[X_i=2]} = 3$.

**Problem 3.** Suppose that $\epsilon_i \sim \text{Binom}(1, 2/n)$ is an iid sequence for $i = 1, 2, \ldots$ and $X_i,\ i = 1, 2, \ldots$ be another, indepndent iid sequence of discrete random variables independent of $N(t)$, taking respective values $0, 1, 2$ with probabilities $0.2, 0.5,$ 0.3. Find

$$\lim_{n\to\infty}\ P\Big(\sum_{j=1}^{3n} I_{[X_j=1]}\,\epsilon_i\ = 5,\ \sum_{j=1}^{2n} I_{[X_j=2]}\,\epsilon_i\ =\ 2\Big)$$

**Problem 4.** Suppose that $N(t)$ is a Poisson process with rate $\lambda$, and let $T_k = \min\{t:\ \ N(t) = k\}$ for $k \geq 1$. Find $E\Big[\ \sum_{j=1}^{n} T_j^2\ \Big|\ N(t) = n\ \Big]$.