

SYLLABUS for Stat 650 MID-TERM:
Chps. 1–2 and Secs. 4.1–4.3 of Durrett.

TOPICS

- (1) Calculation using conditional expectations, including:
 - (a) “First-step analysis”,
 - (b) Calculations of probability and moment generating functions for random sums such as Branching or compound Poisson processes.

- (2) Definition and basic consequences of Markov Property and Strong Markov property for discrete-state and discrete-time chains, including:
 - (a) Chapman-Kolmogorov equation,
 - (b) calculation of finite-time and limiting joint distributions for Markov-chain variables, e.g.: $\lim_{n \rightarrow \infty} P_i(X_n = j, X_{n+m} = k)$.

- (3) Probabilities of absorption and expected time to absorption for discrete time chains, using first-step analysis.

- (4) Classification of states in discrete-time HMC. Checking irreducibility, aperiodicity, recurrence. Equivalent conditions for (each of) irreducibility, aperiodicity, recurrence and positive-recurrence.

- (5) Stationary probability distributions for finite-state discrete- and continuous-time chains. Criteria for positive recurrence including those involving $E_x(T_x)$.

- (6) Calculation of long-term average quantities for discrete-time Markov chains, when expressed in terms of functions on cycles between successive visits to some state.

- (7) Construction and interpretation of continuous-time Markov chains using embedded chains and exponential holding times.

- (8) Equivalent definitions of Poisson processes, and theorems on superposition, thinning and locations of known numbers of jumps.