Stat 700, Fall '01 12/12/01

Sample Problems for Stat 700 Final

The test will be closed-book, but you are permitted to bring 1 or 2 notebook sheets of formulas and notes for references if you want. You are allowed to use calculators if you want.

The following problems are adapted from past finals, qualifying exams, take-homes, etc. and are roughly at the same level of difficulty as the problems you can expect on the Stat 700 Final at 5-7pm on Monday, December 17, 2001. These are for practice only ! Use the outline given in class as a more reliable guide of what to study !!

(1). Tell briefly why each of the following statements is true or false. If false and you can give a corrected statement, do so.

(a). When it exists, the UMVUE for a scalar parameter ϑ minimizes $E_\vartheta(T-\vartheta)^2$ over all parameter-values ϑ and *all* (sufficiently regular) estimators T .

(b). MLE's in full exponential families exist and are unique.

(2). Let V_i for $i = 1, \ldots, n$ be a sample of $Expon(1/\lambda)$ variables (which in my parameterization have mean λ), and let $U_i = [V_i]$ be defined as the greatest integer less than or equal V_i , i.e., $U_i = k$ whenever $k \le V_i < k+1$ for $k \ge 0$. Find the exact Cramer-Rao lower bounds for unbiased estimators of $e^{-\lambda}$ based respectively on samples $\{V_i\}_{i=1}^n$ and $\{U_i\}_{i=1}^n$. Note that you need not find the UMVUE's to do this problem.

(3). Find the UMVUE of ϑ^2 based on a sample of observations $U_1, \ldots, U_n$ from a $Unif(0,\vartheta)$ distribution.

(4). Suppose that $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ are a sample of 2-vectors with $\mathcal{N}(\mathbf{0}, \sigma^2 I_2)$ distributions, where $\mathbf{0} = (0,0)$ and I_2 is the 2×2 identity matrix. Show that $(X_1/Y_1), \ldots, (X_n/Y_n)$ is an *ancillary* statistic, and conclude (justifying all steps) that it is independent of $\sum_{i=1}^n (X_i^2 + Y_i^2)$.

(5). Find the probability distribution of $\sum_{i=1}^n W_i^\alpha$, where W_i are *iid* random variables with *Weibull* density $\lambda\, \alpha\, w^{\alpha-1} \exp(-\lambda\, w^\alpha)\, I_{[w>0]}$, where λ, $\alpha > 0$ are parameters.

(6). Suppose that a data-sample $\{Y_i : 1, \ldots, 2n\}$ is such that $Y_1, \ldots, Y_n$ are *iid* with $Gamma(2, 1/\lambda)$ distribution (density $\lambda^{-2} x \exp(-x/\lambda)$ for $x > 0$) and $Y_{n+1}, \ldots, Y_{2n}$ are *iid* with $Gamma(2, 2/\lambda)$ distribution.

(a) Find the Maximum Likelihood estimator $\hat{\lambda}$ and Method-of-Moments estimator $\tilde{\lambda}$ for the unknown parameter λ based on these data.

(b) Are these estimators of λ unbiased ? Give the UMVUE for estimating λ based upon these data.

(7). *Look over the hierarchical and prior-posterior problems (not necessarily conjugate ones) which we did especially in Chapter 4. These problems are worht reviewing/practicing.*

(8). *Look again at the problems relating* t, F, χ^2 *distributions based on normal samples. The definitions of these distributions are facts which simplify many transformation calculations relating to normal distributions.*