STAT 700 September 19, 2024

Third Problem Set in STAT 700, F24

These 6 problems are due on Wednesday, October 2, 2024, 11:59pm, as a pdf-format upload to the course ELMS page. They include:

(I) Two problems on **Bayes posterior calculation**: # 1.2.4 in Bickel and Doksum, plus:

(2.) Suppose that a random group-label $L = 1, 2$ and random waiting-time T until failure are jointly distributed according to:

$$P(L = 1) = 1 - P(L = 2) = p \ , \qquad f_{T|L}(t|k) \ = \ \lambda_k \, e^{-\lambda_k \, t} \, I_{[t \geq 0]}$$

where $0 < \lambda_1 < \lambda_2 < \infty$ are (known) constants.

(a). Find $P(L = 1 \mid T \geq t)$ as a function of $t > 0$.

(b). Show that the (unconditional) *hazard-rate* $f_T(t)/(1 - F_T(t))$ is a strictly decreasing function of t, while it is a well-known fact that the hazard rates of exponential random variables are constant. Does part (a) help you interpret this fact ?

(II) One problem on minimal sufficient statistics:

(3.) Let X_i for $i = 1, \ldots, n$ be *iid* discrete random variables with values $1, \ldots, k$ (for k known and finite) with probability mass functions given for $1 \leq j \leq k$ by

$$P_\theta(X_i = j) = p_j(\theta) \ = \ c_j \, (a_j + b_j \theta) \ , \qquad \text{where} \qquad \sum_{j=1}^{k} p_j(\theta) \equiv 1$$

where a_j, b_j, c_j are known and θ is unknown. Assume θ is restricted so that all $p_j(\theta) > 0$ and assume $b_j c_j \neq 0$ for at least one value of j. Note that there is still an interval of unknown possible values for θ as long as $\sum_{j=1}^{k} c_j b_j = 0$, which you can assume in doing this problem. What is the (vector) minimal sufficient statistic? (Verify minimality.) Can you guess what is the necessary condition for this minimal sufficient statistic to be complete ?

(III) Another problem on calculation with conditional distributions:

(4.) Suppose that X, Y are bivariate normal with $E(X) = 1, \ E(Y) = 2$, $\text{Var}(X) = 1, \ \text{Var}(Y) = 4$, and $\text{Corr}(X, Y) = 0.5$. Find $E(Y^4 \mid X)$.

(IV) Two problems on UMVUE's:

(5.) Find the UMVUE for $\theta e^{-\theta}$ based on *iid* data $Y_1, \ldots, Y_n \sim \text{Poisson}(\theta)$.

(6.) Find the UMVUE of θ^2 based on *iid* data $U_1, \ldots, U_n \sim \text{Uniform}(0, \theta)$.