

Fourth Problem Set in STAT 700, F24

These 6 problems are due on Friday, October 18, 2024, 11:59pm, as a pdf-format upload to the course ELMS page. They include:

(I) *Two problems on Bayes posterior calculation:* Bickel & Doksum # 1.2.12, 1.2.14.

(II) *Three problems on Decision Theory:* Bickel & Doksum # 1.3.2, and # 1.3.6 (a), (b), and (e) . These 3 parts of #1.3.6 count altogether as 2 problems.

(III) Let $X \sim \text{Binom}(10, \theta)$, where $\theta \in (0, 1)$ is an unknown parameter.

(a) If $\pi(\theta) \propto \theta^2(1-\theta)$ is assumed as the prior density, then find the Bayes estimator of $1/\theta$ with respect to the square-error loss function $L(a, \theta) = (a - \theta^{-1})^2$ where the actions $a \in \mathcal{A} = (1, \infty)$.
Hint in (a): find the posterior density $\pi(\theta | X = k)$.

Note that Bayes estimators are generally not unbiased.

(b) Prove that in this problem, there is no unbiased estimator of $1/\theta$.