STAT 700 October 26, 2024

Fifth Problem Set in STAT 700, F24

These 7 problems are due on Friday, November 8, 2024, 11:59pm, as a pdf-format upload to the course ELMS page. They include:

(1)–(5) Bickel & Doksum # 1.6.5, 1.6.7, 1.6.11(e)–(f), 1.6.16, and 1.6.22

(6) [*Counts as 2 problems*] An observation is made for a discrete random variable $Y \in (0, 1, 2)$ with the probability mass function given θ defined by

$$p(y \,|\, \theta) \;=\; \frac{1}{3}\, I_{[\theta=1/2]} \;+\; \frac{2}{3}\, I_{[y=0,\, \theta=3/4]\cup[y=2,\, \theta=1/4]} \;+\; \frac{1}{6}\, I_{[\theta=1/4,\, y<2]\cup[\theta=3/4,\, y>0]}$$

and

$$\mathcal{A} = \Theta = \{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\} \;\;, \quad L(a,\theta) \;=\; |a-\theta| \quad , \qquad \text{prior} \quad \pi(\theta) \equiv \frac{1}{3} \quad \text{for} \quad \theta = \frac{1}{4},\, \frac{1}{2},\, \frac{3}{4}$$

(a). Find $p_Y(y)$ and $p_{\theta|Y}(\theta|y)$ in the Bayesian setting.

(b). Show that there are 27 pure strategies; that the probability laws on $\mathcal{A}$ can be represented as a closed 2-dimensional set Δ_3 (closed simplex) in $\mathbb{R}^3$; and therefore that the mixed strategies can be represented as a threefold cartesian product $(\Delta_3)^3$.

(c). Argue that the set of triples $(R(\tilde{a},\, \theta),\; \theta \in \Theta) \in \mathbb{R}^3$ form a closed convex set K and that its minimal points (called the lower-left envelope) with respect to the componentwise partial order $\leq$ form a subset of the boundary of K that corresponds to the set of *admissible* mixed strategies for the decision problem.

(d). Find the Bayes optimal stratgy $\tilde{a}^\pi$ for the prior $\pi(\cdot)$ assigning equal mass 1/3 to each $\theta \in \Theta$.

(e). Find the three risks $R(\tilde{a}^\pi,\, \theta)$, for $\theta \in \Theta$.

(f). Find a prior π^* on Θ that has the property that its Bayes estimator $\tilde{a}^{\pi^*}$ has $R(\tilde{a}^\pi,\, \theta)$ equal for all $\theta \in \Theta$. Prove that this $\tilde{a}^{\pi^*}$ must be a minimax strategy for this decision problem in the class of all mixed strategies. Is it unique (as a minimax mixed strategy) ?