Major Topics & Problem Types for In-Class Test, 10/29/2022

The in-class test on Wednesday, November 2, 2022 will consist of three problems. (I might give a fourth problem and offer you a choice of 3.) The test is closed-book but you may bring a single-sided formula sheet as a memory aid if you like.

The major topics that we covered that may figure on the test are:

(I) Statistical models, identifiable parameters, consistent estimation of parameters. Examples including mixtures of distributions.

(II) Decision theory, risk, Bayes risk, Bayes and minimax proedures with respect to a loss function, admissibility, randomized decision rules. Relation between decision-theory properties and set of feasible risk values $R(\theta, \delta)$ for statistical procedures (non-randomized or not).

(III) Sufficient Statistics, Factorization Theorem, minimal sufficiency, complete sufficiency, Rao-Blackwell Theorem, UMVUEs.

(IV) Exponential Families: definitions, natural parameter and parameter space, properties of sufficient statistics and parameterization under canonical exponential families when natural parameter space is open.

(V) Bayesian statistical problems, Bayesian decision theory, conjugate priors, admissibility of Bayes-optimal statistical procedures. Relation between prediction and Bayes-optimal statistical procedures (posterior expected loss minimization).

Examples of Problem Types

(a) Verification of identifiablility or non-identifiability of parameter in statistical problem (with respect to specified models and parameter space. Verification that a parameter can be estimated consistently.

(b) Proof of property like admissibility or minimaxity of statistical procedure in example. Finding Bayes-optimal or minimax procedure in example.

(c) Checking sufficiency, minimal- or complete- sufficiency in example. Finding UMVUE when sufficient statistic is checked to be complete.

(d) Checking exponential family property, natural parameter space, etc. Verification from first principles in example that canonical sufficient statistic is minimal.

(e) Finding conjugate prior in exponential family example; using it to find Bayes-optimal estimator. Checking whether exponential family is curved or (after 1-to-1 reparameterization) canonical.

Specific Examples of Problems

Problems **(2)** and **(5)** from the sample-test 10/12/01 are examples of problems that might be asked. In (5), is $|\lambda - \mu|$ identifiable from the observation $Y - X$ alone, or from a sample of such observations $Y_i - X_i, \;\; i = 1, \ldots, n$?

Problems **(1)**, **(3)** and **(4)** from the in-class test in Fall 2009 are all reasonable problems within the scope of our Fall 2022 in-class test, although a little too long. (The test given in Fall 2009 was 1 hour and 20 minutes, while ours will be 55 minutes.)

Problems **(3)**, **(4)** from the sample problems for the 2009 in-class final are examples of problems that might be asked.

Problems **(F14.7)**, **(F14.9)** might be asked. We covered material related to **(F14.10)** but did not emphasize it heavily, and I would not ask that one this time.

Try to make sure that you understand the decision theory definitions well enough to know how to prove admissibility of Bayes procedures. Also, here is an example of two admissibility proofs you should be able to do quickly from the basic definitions:

Problem (A). In a decision-theory estimation problem with squared-error loss, prove that a UMVUE is an admissible estimator.

Problem (B). Suppose that in a statistical decision problem the minimax decision rule δ is unique. Then prove that δ is admissible.

Problem (C). Give an example of a statistical decision problem in which there is no nonrandomized minimax decision rule or admissible decision rule. (*Hint: you can find a simple example where there is no data* $\underline{\mathrm{X}}$.)