

## Extra STAT 701 Homework Problems, Spring 2025

These are assigned problems, one or two per problem set, that are not in any of our books. They should be done as part of the assigned HW sets with indicated due-dates.

### HW1, due 2/8/2025

(A). Suppose that independent identically distributed observations  $X_1, X_2, \dots, X_n$  have the Gamma(3,  $\theta$ ) density

$$f(x, \theta) = 0.5 \theta^3 x^2 e^{-\theta x} I_{[x>0]}$$

for unknown parameter  $\theta > 0$ .

(a). Find the maximum likelihood estimator  $\hat{\theta}$  of  $\theta$  and the variance  $V_1$  of the large-sample limiting distribution of  $\sqrt{n}(\hat{\theta} - \theta)$ .

(b). Find the variance  $V_2$  of the large-sample limiting distribution of  $\sqrt{n}(\tilde{\theta} - \theta)$ , where  $\tilde{\theta}$  is the Method of Moments estimator of  $\theta$  obtained as a constant multiple of  $(n/\sum_{i=1}^n X_i^2)^{1/2}$ .

(c). Compare the asymptotic variances  $V_1, V_2$ , i.e., note which is larger and find  $V_1/V_2$ . (*Your answer should be a constant, not depending on  $n$  or  $\theta$ .*)

### HW2, due 2/23/2025

(B). Consider testing at significance level  $\alpha$  the one-sided hypothesis  $H_0 : \theta \leq \theta_0$  versus  $H_A : \theta > \theta_0$  for *iid* data  $X_1, \dots, X_n$  from the natural exponential family

$$f(x, \theta) = \exp(\theta T(x) - A(\theta)) h(x) \quad , \quad x \in \mathbb{R}$$

for a scalar parameter  $\theta$ . Prove that the UMP test you get from MLR theory and the Karlin-Rubin Theorem is exactly the same as the Likelihood Ratio Test for this hypothesis. Assume that  $T(X_1)$  is continuously distributed, and define each of these tests with its exact rejection threshold for significance level  $\alpha$ .