Eric Slud Spring 2010

Sample Problems for Final, Stat 701

These problems are at about the level that will be asked on the exam, but they do not cover the full range of things that might be asked. See the old sample exams and tests for further examples. 5 or 6 problems of 20 points each will be asked on the exam, with 100 points a perfect score.

Instructins. *Justify your assertions by appeal to theorems from the course wherever possible.*

(I). (*ARE of 2dim GMOM*) Suppose that X_i, Y_i for $i = 1, \ldots, n$, are all *iid* random variables with $\mathcal{N}(\mu, \sigma)^2$ distribution. Find the asymptotic relative efficiency (versus the MLE) of the estimator of $a = \mu\sigma^2$ defined by $T = \sum_{i=1}^n (X_i + Y_i)(X_i - Y_i)^2/(4n)$. (*Hint: for* $Z \sim \mathcal{N}(0,1)$, $E(Z^4) = 3$.) What is the MLE of a ?

(II). (*Pivotal quantity, CI*) Suppose that $Z_1, \ldots, Z_n$ are a large sample of double-exponential observations, with density

$$f(z, \mu, \sigma) = (2\sigma)^{-1} \exp(-|z - \mu|/\sigma) \qquad z \in \mathbf{R}, \qquad \mu \in \mathbf{R}, \ \sigma > 0$$

Find an approximately 95% two-sided confidence interval for σ based on the pivotal quantity s_Z^2/σ^2 where $s_Z^2 = (n-1)^{-1} \sum_{i=1}^n (Z_i - \bar{Z})^2$.

Problem (III) on the Spring 2009 Sample Final is similar in spirit.

(III). (*Min Contrast estimator, checking uniqueness & consistency*) Suppose that independent identically distributed random pairs (X_i, Y_i) satisfy

$$\epsilon_i \equiv Y_i - a - bX_i \qquad \text{is symmetric and indep. of} \quad X_i \ , \quad \text{and} \quad E(\epsilon_i^4) < \infty$$

Show that $\rho(\mathbf{X}, \mathbf{Y}, a, b) = \sum_{i=1}^n (Y_i - a - bX_i)^4$ is a contrast function, and show that the estimator $(\tilde{a}, \tilde{b})$ defined by minimizing $\rho(\mathbf{X}, \mathbf{Y}, a, b)$ is uniquely defined and consistent. (*Hint: expand the monomials, after subtracting and adding* $a_0 + b_0 X_i$ *inside them.*)

(IV). (*UMP Test, exponential family asymptotic cutoff*) Find the approximate large-sample rejection region and power against $\gamma = 1.2$ for the level $\alpha = .05$ UMP test of $H_0 : \gamma \le 1$ versus $H_A : \gamma > 1$, based on a data-sample $X_1, \ldots, X_{100}$ from the density $f(x, \gamma) = (x/\gamma) \exp(-\frac{1}{2} x^2/\gamma) I_{[x>0]}$, where the unknown parameter γ is positive.

(V). (*GLRT using Wilks*) Find the Likelihood Ratio Test statistic with approximate large-sample cutoff for testing $H_0 : p_1 = 2p_3$, against the general alternative, based on multinomial data $(n_1, n_2, n_3) \sim \text{Multinomial}(n, p_1, p_2, p_3)$.

(VI). (*Bias correction of an estimator*) Let $Y_n \sim \text{Poisson}(n\,(1+3\lambda))$. Find smooth functions $g(\cdot)$ and $h_n(\cdot)$ with the properties that $g(Y_n/n)$ has asymptotic variance not depending on λ, and that $h_n(Y/n)$ has bias $o(1/n)$ as an estimator of $\sqrt{\lambda}$.

(VII). (*Test-based CI's following from Rao Score*) Suppose that you observe a large data-sample from the density

$$f(x,\vartheta) \;=\; 3\,\vartheta\,x^2\,\exp(-\lambda x^3)\,I_{[x>0]}$$

Find the (approximate large-sample) Test-based confidence interval derived the family of Rao-Score tests of hypotheses $H_0 : \vartheta = \vartheta_0$.