Eric Slud, STAT 730 January 26, 2026

HW1 for STAT 730, Spring 2026

Instructions: Do all problems, which count equally (10pts.), except that Problem 2 counts double (20pts.) Show your reasoning and give a textbook reference for any formulas or facts you use.

This problem set is due 11:59pm Thursday Feb. 12, to be uploaded as `*.pdf` or `*.doc` on ELMS.

Do #1.13 (assuming that w_t is **Gaussian** white noise) and #1.14 from Shumway and Stoffer (2nd or 3rd ed.), plus the following four problems.

Problem 1. Prove the following fact from first principles (i.e., from definitions and basic properties), citing all basic properties you use in order to make the proof as self-contained as possible. Suppose that $(X_1, \ldots, X_K)$ is a random Gaussian vector with nonsingular $K \times K$ covariance matrix V. Prove that for any two distinct indices $i,\, j \in \{1, \ldots, K\}$, the random variables X_i, X_j are conditionally independent given $\{X_a : \; 1 \le a \le K, \; a \ne i, \; a \ne j\}$ if and only if $(V^{-1})_{ij} = 0$.

Problem 2. (cf. Problems 1.9 and 1.16 in Shumway and Stoffer; this one counts as 2 problems.) Let ξ_1, ξ_2 be independent identically distributed random variables with means 0 and variances σ^2, and $\eta \sim \text{Unif}(0, \pi)$ be independent of ξ_1, and let $\omega_0 > 0$ be a nonrandom constant. Define the continuous-time random periodic functions (with period ω_0)

$$Y_t \, = \, \xi_1 \, \sin(2\pi t/\omega_0) \, + \, \xi_2 \, \cos(2\pi t/\omega_0) \;\; , \qquad Z_t \, = \, \xi_1 \, \sin(2\pi \, t/\omega_0 + \eta)$$

(a). Show that both $Y_t, \; Z_t$ are weakly stationary random functions of $t > 0$.

(b). Show also that $Y_t, \; Z_t$ are strictly stationary if $\xi_i \sim \mathcal{N}(0, \sigma^2)$.

(c). But show that Y_t is **not** strictly stationary if ξ_i has double-exponential density $2\, e^{-|x|}$ for $x \in \mathbb{R}$.

Problem 3. Do Shumway-Stoffer #1.2(a) together with # 1.22. *These together count as one problem (10 pts.)* In #1.22, explain what the ACF is telling about the series in #1.2(a).

See next page for one more problem.

Problem 4. Let V_i for $i = 1, 2, \ldots$ be *iid* Expon(λ), and define the Poisson process $N(t)$ with random initial state by $N(0) \sim \text{Binom}(1,\ 1/2)$ and

$$N(t) \;=\; N(0) + \max\{k \geq 0 :\; \sum_{i=1}^{k} V_i \,\leq t\}$$

where the random variables V_i are (jointly) independent of $N(0)$. (This process has the property – which you may use as needed – for all $0 < s < t$, that $N(t) - N(s) \sim \text{Poisson}(\lambda(t-s))$ is independent of $(N(u),\ u \leq s)$.)

(a). Use these definitions/properties to show that $N(t)$ as a continuous-time stochastic process is not weakly stationary, but does have stationary increments which implies it is 'intrinsic'.

(b). Let $Z(t) \equiv I_{[N(t)\ \texttt{is odd}]}$. Prove that $Z(t)$ is strictly stationary.

Hint: the simplest way is to use the memoryless property of the exponential distribution. Alternatively, you may use a suitable Markov property to do your proof, after saying exactly which processes are Markovian, in which case you should give the transition density $p_s(z_1,\, z_2)$ *for* $Z(t)$*.*