

HW2 for STAT 730, Spring 2026

Reading for this HW consists of Shumway-Stoffer Chapter 2, plus Section A.2 and B.1 (Appendices A,B).

Instructions: Do all problems, which count equally (10pts.), except that #2.8 in Shumway-Stoffer counts double (20pts.) Show your reasoning and give a textbook reference for any formulas or facts you use.

This problem set is due 11:59pm Sunday Mar. 1, to be uploaded as *.pdf or *.doc on ELMS.

Do #2.5, #2.8 (counts as 2 problems), #2.9 (this is 2.9 in the 2nd edition and #2.11 in the 3rd edition), plus the following three problems.

Problem (I). Suppose that $w_t \sim (0, \sigma^2)$ for $t \in \mathbb{Z}$ for all integer t is a White Noise series, and that $X_t = \sum_{j=-2}^2 w_{t-j}$, $Y_t = X_{t-1} + X_t + X_{t+1}$.

- (i) Derive $\gamma_X(h)$, $\gamma_Y(h)$, $\gamma_{XY}(h)$.
- (ii) Express Y_t as a Moving Average of w_s .
- (iii) Prove that w_t cannot be expressed as a finite-order moving average of X_t .

Problem (II). Let Y_t be a stationary AR(1) process with $|a_1| < 1$, based on white noise w_t with mean 0 and finite fourth moment. Prove that Y_t is a *mixing* process, in the sense defined in class.

Problem (III). Do #1.28 part (a). (This problem is #1.28 in the 2nd edition, and #1.29 in the 3rd edition.) In place of part (b) as given in the book, characterize the collection of stationary MA(q) processes Z_t (defined in terms of white noise $\epsilon_t \sim (0, \sigma^2)$) for which $\sum_{h=-\infty}^{\infty} \gamma_Z(h) = 0$, and give a completely elementary proof for all such processes Z_t that $E(\sum_{t=1}^n Z_t)^2$ is uniformly bounded for all n .