April 4, 2026

Additional Practice Problems for Test, STAT 730

Instructions: For the test, you can use two-sided $8.5'' \times 11''$ notebook sheet for reference, but no books or other reference materials. You may use a calculator, but will not be required to simplify numerical expressions.

(1). Suppose that $X_t = \phi X_{t+1} + w_t$, for $w_t \sim \mathcal{N}(0, \sigma^2)$, for all $t \in \mathbb{Z}$, where $|\phi| < 1$.

(a). Show that X_t is stationary, with a representation $X_t = \sum_{j=0}^{\infty} g_j \, w_{t+j}$, where $g_0 = 1$.

(b). Does X_t also have a representation $X_t = \sum_{j=0}^{\infty} h_j \, w^*_{t-j}$ for some other white-noise random sequence w^*_t ?

(c) Find the one-step-ahead prediction $\tilde{x}_t^{t-1} = E(x_t \,|\, x_s, \, s \le t-1)$ and corresponding prediction variance $\tilde{P}_t^{t-1} = E((x_t - \tilde{x}_t^{t-1})^2 \,|\, x_s, \, s \le t-1)$.

(2). Suppose that $x_1, \ldots, x_{1000}$ is a realization of a stationary square-integrable linear time series $x_t = \sum_{j=0}^{\infty} \psi_j w_{t-j}$, where $\sum_{j=0}^{\infty} |\psi_j| < \infty$ and w_t is a white-noise with finite 6'th moment. If the raw periodogram (with frequencies scaled to range from $-1/2$ to $1/2$) for the realization yields estimates $\hat{f}(.352) = 3.7$, $\hat{f}(.354) = 2.2$, $\hat{f}(.527) = 1.7$, $\hat{f}(.529) = 4.6$, for the spectral density $f(\cdot)$ find an approximate 90% Confidence interval for $f(0.353)/f(0.528)$. *NB.* $\chi^2_2 \sim$ *Expon*$(1/2)$.

(3). Based on observations X_1, X_2, X_3 of a stationary time series with autocovariances $\gamma(0) = 1$, $\gamma(1) = -4/7$, $\gamma(2) = -3/14$, find the one-step-ahead prediction X_3^2 for X_3 in terms of X_1, X_2 and the corresponding prediction variance.