151a: Applied Numerical Methods. Fall '98

Undergraduate course in Applied Mathematics

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1. Solution of Nonlinear Systems

   1.1 The bisection method

   1.2 Newton's Method

   ... local vs. global convergence, multiple roots

   1.3 Extensions of Newton's method

   ... divided differences: the secant method, Steffensen's method...,

   ... Systems of equations

   ... high-order extensions

   1.4 Fixed point iterations

   1.5 Accelerations

   ... Aitken's process, Steffenssen's method

   1.6 *Polynomial Equations

   ... Local methods – Newton, Bairstow,...

   ... Global methods – Bernoulli, Graeffe, Laguerre, the square root method,...

2. Approximation Theory

   2.1 General overview

   ... On the choice of norm: L^∞ vs. L^2

   ... Weierstrass' density theorem, Bernstein polynomials,

   2.2 Interpolation I. Lagrange interpolant

   2.3 Interpolation II. Newton interpolant

   ... Divided differences. Error estimates.

   ... Equidistant points. Synthetic calculus

   ... Forward backward and centered formulae.

   2.4 Interpolation III. Error estimates.

   ... Runge effect, region of analyticity

   2.5 *Addt'l topics

- Interpolation IV. Interpolation with derivatives
  ... Hermite interpolation; Splines
- Trigonometric interpolation
  ... FFT, truncation + aliasing, error estimates,
  ... elliptic solvers, fast summations (- discrete convolution),...

2.6 Least Squares Approximations I. A general overview
  ... Gramm mass matrix, ill-conditioning of monomials in L^2

2.7 Least Squares Approximations II. Fourier expansion
  ... Bessel, Parseval, ...

2.8 Least Squares Approximations III. Orthogonal polynomials
  ... Examples: Legendre, Chebyshev,...
  ... Sturm's sequence; more examples: Jacobi, Hermite,..

2.9 *Trigonometric polynomials
  ... Complex & real representation; Chebyshev interpolation,..

2.10 MinMax Approximation
  ... Smoothness & Error Estimates

Midterm

3. Numerical Differentiation

3.1 The method of undetermined coefficients
  ... The differentiated algebraic interpolant;
  ... *The differentiated spline interpolant
  ... Error estimates

3.2 Equidistant Differentiation
  ... The synthetic approach – operational calculus
  ... Richardson's extrapolation

3.3 *Addt'l topics
- Trigonometric differentiation
- Spline differentiation

4. Numerical Integration – Quadratures

4.1 Equidistant points
  ... Newton-Cotes formulae
  ... Composite Simpson's rule
  ... Romberg & adaptive integration

4.2 Gauss Quadratures

5. Solution of Linear Systems – Direct Methods

5.1 Introduction

... Cramer's rule costs $N!$; diagonal, triangular and unitary systems

5.2 Gauss elimination

... The basic algorithm

... pivoting, multipliers, update, back substitution

... Operations count

... storage & pivoting for special matrices:

... symmetric, positive definite, banded, Hessenberg,...

5.3 LU decomposition

... Elementary matrices

5.4 Backward error analysis

... norms, a priori estimates, ill-conditioning,

... condition number, backward error estimates

... Pivoting: partial & full pivoting

... amplification factors: diagonally dominant, symmetric pos. definite,

... banded & Hessenberg matrices

... Equilibration: counterexamples,

... equilibration & pivoting – scaling, iterative refinement

5.5 Direct LU decompositions

Crout, Doolittle & Cholesky, tridiagonal systems

5.6 *Addt'l topics

- Other direct methods
  ... Cayley-Hamilton, Conjugate Gradient, block decomposition,
  ... Sherman-Woodbury formula,QR decomposition
- Special Fast Solvers
  ... FFT, Circulant matrices, Toeplitz,...

Review

References

[At] K. Atkinson, *An Introduction to NUMERICAL ANALYSIS*, Wiley, 1987.

[CdB] S. Conte & C. deBoor, *ELEMENTARY NUMERICAL ANALYSIS*, McGraw-Hill, User friendly; Shows how 'it' works; Proofs, exercises and notes

[DB] G. Dahlquist & A. Bjorck, *NUMERICAL METHODS*, Prentice-Hall, User friendly; Shows how 'it' works; Exercises

[IK] E. Isaacson & H. Keller, *ANALYSIS of NUMERICAL METHODS*, Wiley, The 'First'; Proofs; out-dated in certain aspects; Encrypted message in Preface

[RR] A. Ralston & P. Rabinowitz, *FIRST COURSE in NUMERICAL ANALYSIS*, 2nd ed., McGraw-hill, Detailed; Scholarly written; Comprehensive; Proofs exercises and notes

[SB] J. Stoer & R. Bulrisch, *INTRODUCTION TO NUMERICAL ANALYSIS*, Springer-Verlag, detailed account on approximation, linear solvers & eigensolvers, ODE solvers,..

[W] B. Wendroff, *THEORETICAL NUMERICAL ANALYSIS*, Academic Press, 1966, Only the 'Proofs'; elegant presentation