Control of Stochastic Behaviors in Robotic Swarms using PDE Models

Karthik Elamvazhuthi, Spring Berman

Arizona State University: Mechanical and Aerospace Engineering

Problem Statement

We study a task allocation problem for a swarm of robots where a target distribution of spatial coverage by the swarm is desired. We model the population dynamics of the robots using a set of advection reaction diffusion partial differential equations (PDEs). The task allocation problem is then framed and solved as an optimal control problem.

Microscopic Model

- Agent primitives based on stochastic differential equation formalism

- Robots' changes in state are modeled as a Chemical Reaction Network (CRN) in which the species are *F*, a flying robot; H_j, a robot that is hovering over a flower of type *j*; and Vj, an instance of a robot visit to a flower of type *j*

$$F \xrightarrow{k_j(t)} H_j + V_j$$

$$H_j \xrightarrow{k_f} F$$

- Robot i has position $\mathbf{x}_i(t) = [x_i(t)\ y_i(t)]^T$ at time t.

- Time-dependent velocity field $\mathbf{v}(t) = [v_x(t)\ v_y(t)]^T$

- Robots' motion over time step Δt modeled using the standard-form Langevin equation:

$$\mathbf{x}_i(t+\Delta t) - \mathbf{x}_i(t) = \mathbf{v}(t)\Delta t + (2D\Delta t)^{1/2}\,\mathbf{Z}(t)$$

Macroscopic Model

- $\Omega \in \mathbb{R}^2$ is an open bounded subset with Lipschitz continuous boundary $\partial\Omega$. $Q = \Omega \times (0,T)$ and $\Sigma = \partial\Omega \times (0,T)$.
$\vec{\mathbf{n}} \in \mathbb{R}^2$ is the outward normal to $\partial\Omega$.

- There are n_f types of flowers.

- $H_i : \Omega \to \{0,1\}$ are indicator functions that model the presence or absence of flower type i over the domain Ω.
- $y_1(\mathbf{x},t), y_2(\mathbf{x},t)$ and $y_3(\mathbf{x},t)$ are the states representing the density fields of flying robots, hovering robots and flower visits at $(\mathbf{x},t) \in Q$.

$$\frac{\partial y_1}{\partial t} = \nabla\cdot(D\nabla y_1 - \mathbf{v}(t)y_1) - \sum_{i=1}^{n_f} k_i H_i y_1 + k_f y_2 \quad in\ Q,$$

$$\frac{\partial y_2}{\partial t} = \sum_{i=1}^{n_f} k_i H_i y_1 - k_f y_2 \quad in\ Q,$$

$$\frac{\partial y_3}{\partial t} = \sum_{i=1}^{n_f} k_i H_i y_1 \quad in\ Q, \qquad \vec{\mathbf{n}}\cdot(D\nabla y_1 - \vec{\mathbf{v}}(t)y_1) = 0$$

$$y(0) = y_0$$

Optimal Control Problem

$$\min_{(y,u)\in Y\times U_{ad}} J(y,u) = \frac{1}{2}\|Wy(\cdot,T) - y_\Omega\|^2_{L^2(\Omega)^{1+n}} + \frac{\lambda}{2}\|u\|^2_{L^2(0,T)^m}$$

$$U_{ad} = \{u \in L^2(0,T)^{m+2};\ u_i^{min} \leq u_i \leq u_i^{max}\ \ a.e.\ \ in\ \ (0,T)\}$$

$$\vec{v} = (u_1,u_2) = \vec{u}_b,\ u_i = k_{i-2} \text{ for } 3 \leq i \leq m+2 \text{ and } m = n_f$$

- **Unique weak solution exists** satisfying the PDE as an equality in $Y^* = L^2(0,T;X^*)$. Here $X = V \times L^2(\Omega)^2$ and $V = H^1(\Omega)$.

- **Existence of optimal control** can be proven using standard arguments based on weak compactness of closed bounded sets in the corresponding infinite dimensional spaces and embedding arguments.

- **Directional Derivatives** of the control to state map exist along directions $h \in L^\infty(0,T)^{m+2}$.

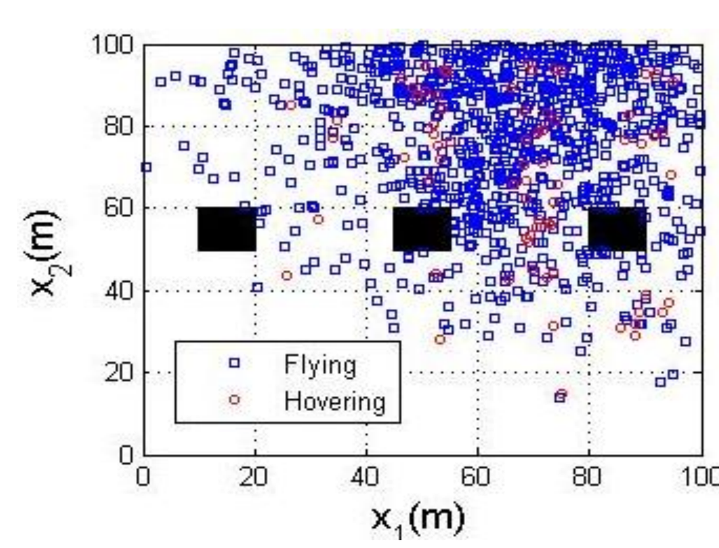
- **Adjoint equation** characterizes the first order necessary conditions for the optimal control:

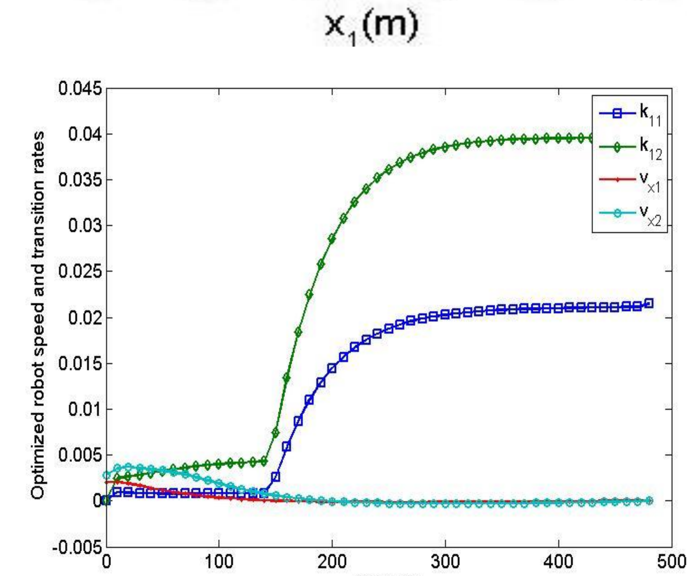
$$-\frac{\partial p_1}{\partial t} = \nabla\cdot(D\nabla p_1 + \mathbf{v}(t)p_1) + \sum_{i=1}^{n_f} k_i H_i(-p_1 + p_2 + p_3) \quad in\ Q,$$

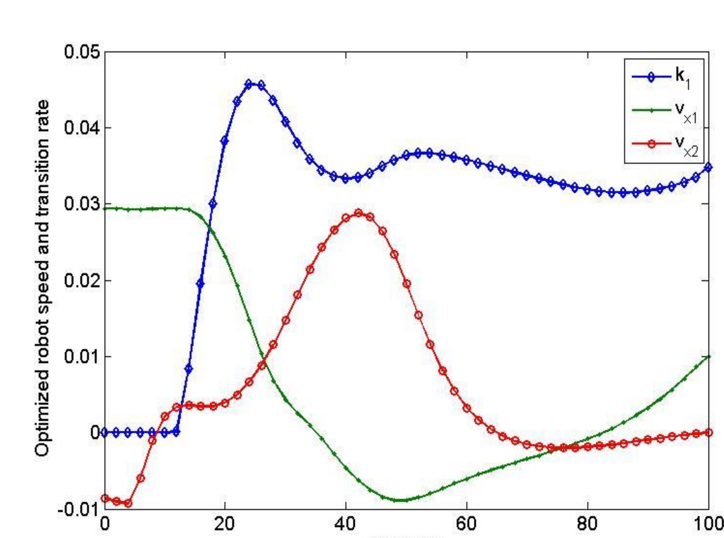
$$-\frac{\partial p_2}{\partial t} = k_f p_1 - k_f p_2 \quad in\ Q, \qquad \vec{n}\cdot\nabla p_1 = 0 \quad on\ \Sigma$$

$$-\frac{\partial p_3}{\partial t} = 0 \quad in\ Q, \qquad p(T) = W^*(Wy(\cdot,T) - y_\Omega)$$

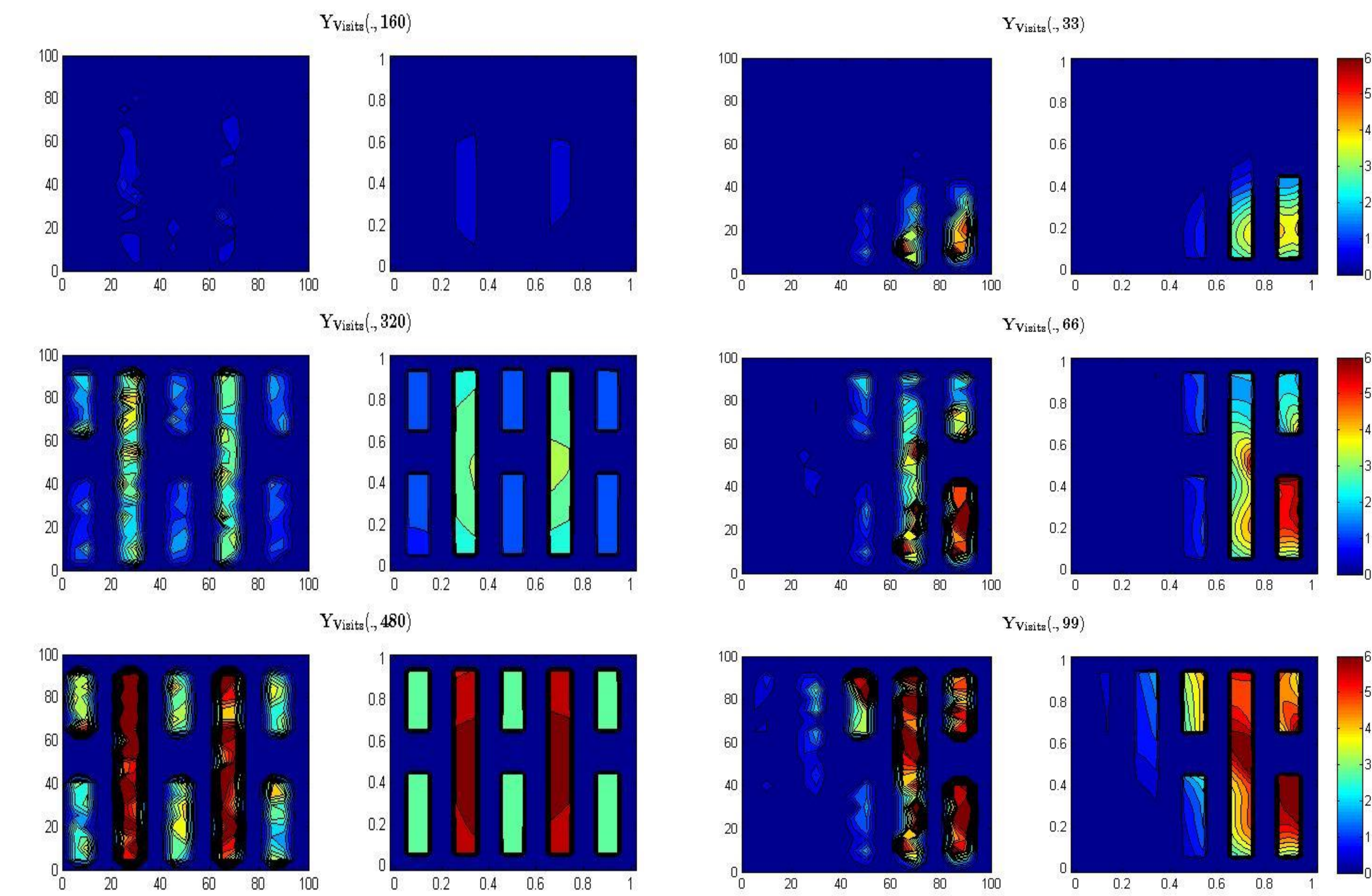
Simulation Results





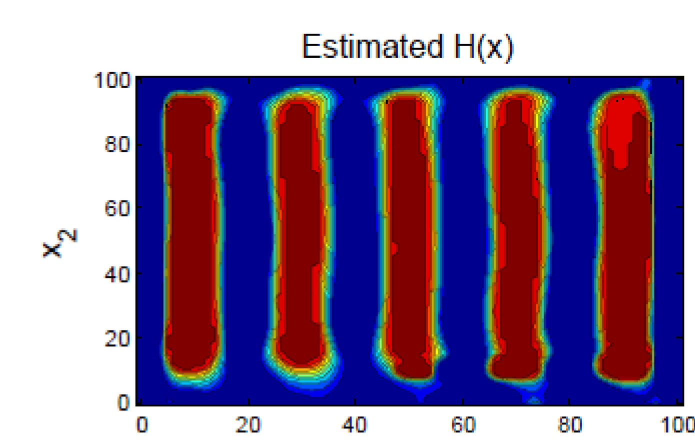


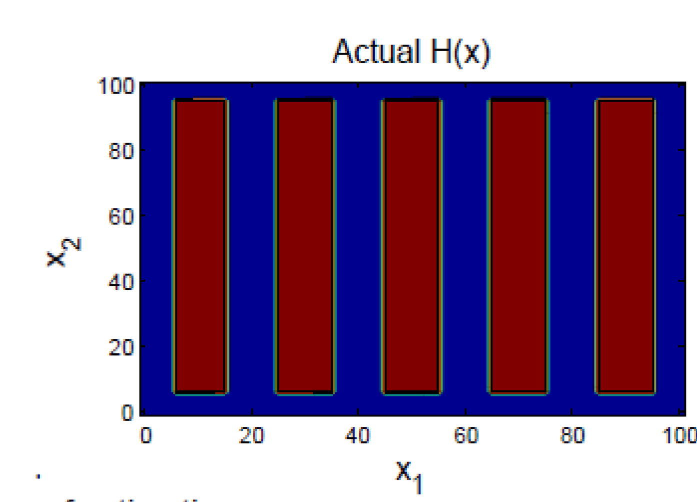
The **top-left** figure is a snapshot of the robot states for a sample test case.
The **bottom-left** figure shows the optimized control parameters for a test case with two flower types and a spatially non-uniform target distribution of robot activity.
The **bottom-right** figure shows the optimized control parameters for a test case with one flower type and uniform target distribution of robot activity over the two rightmost rows

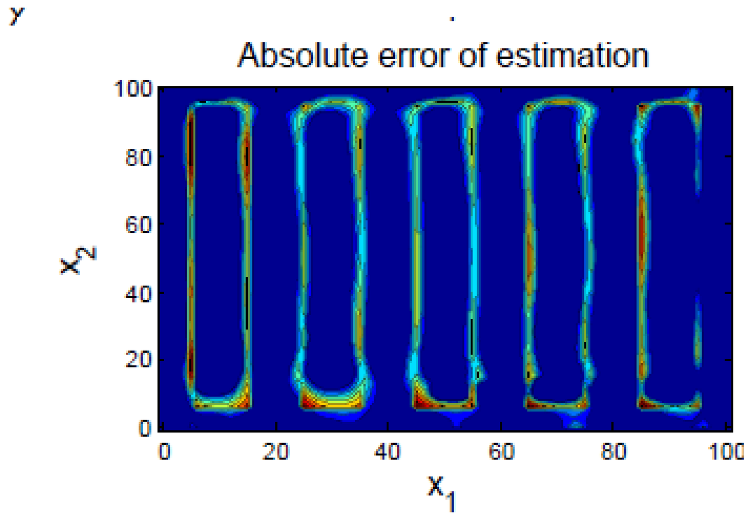


Results of the two test cases from microscopic and macroscopic simulations. The first case **(left)** corresponds to a non-uniform target distribution. The second case **(right)** requires only the two rightmost crop rows to be pollinated.

This framework can also be used to map features of interest when the task spatial distribution (indicator functions H_i) is not known a priori [2]







[1] K. Elamvazhuthi and S. Berman, "Optimal control of stochastic coverage strategies for robotic swarms", in Proc. Int'l. Conf. on Robotics and Automation (ICRA), Seattle, WA, 2015, Accepted.

[2] K. Elamvazhuthi, "A variational approach to planning, allocation and mapping in robot swarms using infinite dimensional models," Master's thesis, Arizona State University, 2014.