Quiz 1 – 02/02/05

Math 241H

Name:

1. (a) (4 points) Find a vector that is perpendicular to both $\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ and $2\mathbf{i} + 2\mathbf{j}$.

$$\begin{vmatrix} i & j & k \\ 1 & -3 & -1 \\ 2 & 2 & 0 \end{vmatrix} = i(2) + j(-2) + k(2+6)$$
$$= 2i - 2j + 8$$

(b) (4 points) Find the projection of $\mathbf{b} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ onto $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

$$\begin{aligned} pr_{\underset{\sim}{a}}\underset{\sim}{b} &= \left(\frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{a}\cdot\underset{\sim}{b}}\right)\underset{\sim}{a} &= \left(\frac{2+2-2}{4+4+1}\right)\underset{\sim}{a} \\ &= \frac{2}{9}(2i+2j+k) &= \left(\frac{4}{9}i + \frac{4}{9}j|\frac{2}{9}k\right) \end{aligned}$$

2. Consider the lines ℓ_1 and ℓ_2, defined as follows. The line ℓ_1 contains the points $(2,1,-3)$ and $(0,-1,-1)$ and the line ℓ_2 contains the points $(0,1,1)$ and $(1,2,0)$.
(a) (2 points) Find the vector equations for ℓ_1 and ℓ_2.

$\ell_1 : (0,-1,-1) + t(2,2,-2)$

$\ell_2 : (1,2,0) + t(1,1,-1)$

(b) (4 points) Find the parametric and symmetric equations for ℓ_1.

$\underset{\sim}{P}arametric$	$\underset{\sim}{S}ymmetric$
$x = 2t$	$\frac{x}{2} = \frac{y+1}{2} = \frac{z+1}{-2}$
$y = 2t - 1$	
$z = -2t - 1$	

(c) (6 points) These lines happen to be parallel. Explain why. Also, find the distance between the parallel lines ℓ_1 and ℓ_2.

The lines are parallel because the vectors $\underset{\sim}{L}_1$ and $\underset{\sim}{L}_2$ are constant multiples of each other. Alternatively, we can show that $\underset{\sim}{L}_1 \times \underset{\sim}{L}_2 = 0$

$P_1 = (0,-1,-1)$ lies on ℓ_1

$P_2 = (0,1,1)$ lies on ℓ_2

$\underset{\sim}{L} = (1,1,-2)$

$||\underset{\sim}{L}|| = \sqrt{3}$

$$\begin{aligned} ||\underline{L} \times P_1P_2|| &= \sqrt{16+4+4} \\ &= \sqrt{24} \end{aligned}$$

$$\begin{aligned} D &= \frac{||\underline{L} \times P_1P_2||}{||\underline{L}||} \\ &= \frac{\sqrt{24}}{\sqrt{4}} \end{aligned}$$

3. You are pushing a box up a slope that is inclined at an angle of $\pi/6$ above horizontal.

(a) (5 points) How much work do you do if you push the box 100 feet up the slope, exerting a constant force of 15 lbs. in the horizontal direction? You may ignore the effects of friction.

Let $\underline{u}$ be the vector in the direction of the slope, with length 100. Then:

$$\begin{aligned} \underline{u} = &\ 100(cos(\tfrac{\pi}{6})i + sin(\tfrac{\pi}{6})j) \\ = &\ 100(\tfrac{\sqrt{3}}{2}i + \tfrac{1}{2}j) \\ = &\ 50(\sqrt{3}i + j) \end{aligned}$$

Let $\underline{F}$ be the force applied in the horizontal direction. $\underline{F} = 15i$. Then the work done is:

$$\begin{aligned} \underline{W} = &\ \underline{F} \cdot \underline{u} \\ = &\ 15 \cdot 50\sqrt{3} \\ = &\ 750\sqrt{3} \end{aligned}$$

(b) (5 points) Suppose the slope is frictionless and that the box weights 20 lbs.. Suppose you push the box in the direction parallel to the slope. How much force do you need to exert in order to prevent the box from slipping down the frictionless slope?

$$F = 20cos(\tfrac{\pi}{3}) = 10$$