# Take-Home Quiz 4 – due 03/04/05

## Math 241H

Name:

1. (a) (4 points) 13.4 #30.

   Differentiate by x to obtain $0 = 2xz^2 + x^2 2z\dfrac{\partial z}{\partial x} - 2yz - 2xy\dfrac{\partial z}{\partial x} + 3z^2 y^2 \dfrac{\partial z}{\partial x}$

   $$\frac{\partial z}{\partial x} = \frac{2yz - 2xz^2}{3z^2 y^2 - 2xy + 2x^2 z}$$

   Similarly, $\dfrac{\partial z}{\partial y} = \dfrac{2xz - 2z^3 y}{2x^2 z - 2xy + 3z^2 y^2}$

   (b) (4 points) 13.4 #34.

   Let $w = f(r, s, t)$ where $r = x - y$; $s = y - z$; $t = z - x$.

   $$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial w}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial w}{\partial t}\frac{\partial t}{\partial x} = \frac{\partial w}{\partial r} - \frac{\partial w}{\partial t}$$

   In a similar way, we compute:

   $$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial s} - \frac{\partial w}{\partial r} \qquad and \qquad \frac{\partial w}{\partial z} = \frac{\partial w}{\partial t} - \frac{\partial w}{\partial s}$$

   With this we compute $\dfrac{\partial w}{\partial x} + \dfrac{\partial w}{\partial y} + \dfrac{\partial w}{\partial z} = 0$

   (c) (4 points) 13.4 #38.

   $f(tx, ty) = t^n f(x, y)$ Write $r = tx$ and $s = ty$

   $$\frac{\partial f}{\partial t}(r, s) = x\frac{\partial f}{\partial r} + y\frac{\partial f}{\partial s} = nt^{n-1} f(x, y)$$

   Putting $t = 1$ gives $xf_x(x, y) + yf_y(x, y) = nf(x, y)$

2. (8 points) 13.5 #16.

$$\nabla f(x, y) = (w + wxy, x^2 + sin(y) + ycos(y)$$

a) $D_{\vec{u}} f(1, 0) = \nabla f(1, 0) \cdot \vec{u} = (2, 1) \cdot (a, b) = 2a + b$
(subject to $a^2 + b^2 = 1$).
b) $D_{\vec{u}} f(1, 0)$ is maximal when $(a, b)$ is parallel to $\nabla f(1, 0)$.
That is, when $(a, b) = (\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$

3. (a) (5 points) 13.6 #62.

$$z = 5 - x^2 - 2y^2 = f(x, y) \text{ is the altitude at } (x, y).$$

$$\nabla f(x, y) = (-2x, -4y)$$

$$\nabla f(1/2, -1/2) = (-1, 2)$$

(b) (5 points) 13.6 #64. (Use the back if necessary.)