Some problems related to Stoke's and Divergence theorems

Math 241H

1. (a) Show by direct calculation that the divergence theorem does not hold for $\mathbf{F}(r, \theta, \psi) = \frac{\mathbf{r}}{r^2}$, where $\mathbf{r}$ denotes the unit radial vector. Why does the theorem fail?
(b) Verify by direct calculation that the divergence theorem does hold for the $\mathbf{F}$ from part (a) when S is the surface S_1 of a sphere of radius R_1 plus the surface S_2 of a sphere of radius R_2, both centered at the origin, and D is the region between the two surfaces?
(c) In general, what restriction must be placed on a surface S so that the divergence theorem will hold for the function of part (a)?

2. Use the divergence theorem to show that

$$\int \int_S \mathbf{n}\, dS = 0$$

where $\mathbf{n}$ is the unit vector normal to the surface S.

3. Let S be the surface of the sphere $x^2 + y^2 + z^2 = 9$. Evaluate

$$\int_S x^2\, dy\, dz + y^2\, dz\, dx + z^2\, dx\, dy$$

using the divergence theorem.