Matlab 3 [1], Due Nov. 30

Math 340, Fall 2006

Using Matlab 6.5 or newer for Numerical Evaluation of Multiple Integrals

Very few functions have integrals which can be written down as a formula involving well-known functions. Yet integration is important, so what do you do if you need to know the integral of something but you can't find an antiderivative? You must turn to numerical techniques. A numerical technique will provide you with a number which usually is a good approximation of the integral.

Single integrals You can use the matlab function `quad` which allows you to perform a single integration. For example, to evaluate $\int_2^4 \sin(x^3 - 7x)\,dx$ you could type:

```
>> f = inline(vectorize('sin(x^3 - 7*x)'),'x')
>> quad(f,2,4)
```

Matlab has a second numerical integrator called quadl which uses a different method from quad. As a rough check on accuracy you could integrate using quadl also by typing `quadl(f,2,4)` and compare the answers to see if they agree closely enough. There are ways to specify how accurate you would like quad or quadl to be. Type `help quad` to find out how. Generally, you need not worry about this and just use default values.

Newer versions of Matlab are discouraging the use of inline functions as used above and instead advocate using either m-files or anonymous functions. Here's how you would do the above example with anonymous functions.

```
>> f = @(x) sin(x.^3 - 7.*x)
>> quad(f,2,4)
```

The `@(x)` says it is a function of a single variable `x` and the formula follows. You must vectorize each function before integrating numerically. Unfortunately I am not aware of a simple way to vectorize an anonymous function, so you will have to do this by hand or else do a separate vectorize statement and then cut and paste. Vectorizing just means placing a period just before each `*`, `/`, or `^`. Examples below use anonymous functions which may not be present in Matlab 6.5. Or you could stick with inline functions. Just be sure to include your function in single quotes or vectorize may complain.

Double Integrals

Matlab has a command `dblquad` which allows you to automatically evaluate a double integral over a rectangular region. For example, to evaluate $\int_1^2 \int_3^4 \sqrt{x^2 + 4/y^2}\,dx\,dy$ type:

```
>> f = @(x,y) sqrt(x.^2+4./y.^2)
>> dblquad(f,3,4,1,2)
```

[1]Thanks to my colleague Henry King for the file for this assignment.

Note that the limits of the x variable come first, then the limits of the y variable since this is the order the variables were specified in the anonymous function f.

If the region R you are integrating over is not rectangular, then you have a number of options.

Among them are:

1) If the region is vertically or horizontally simple, you might be able to do the inner integration by hand if you are lucky. Then you have reduced the problem to a single integral which you can evaluate using quad.

2) You could set the function equal to zero outside R and use dblquad.

3)If the region is vertically or horizontally simple, you could use an mfile `ezint241.m`, available on Prof. Henry King's website, which will integrate for you.

A sample evaluation of a double integral Let us consider the case where R is the triangular region $\{(x,y) \mid x \geq 0, y \geq 0, x+y \leq 1\}$ and use methods 1 and 2 above to find $\int_0^1 \int_0^{1-y} \sin(xy)\,dx\,dy$. For the first method, note that

$$\int_0^{1-y} \sin(xy)\,dx = -(\cos(xy))/y\Big]_0^{1-y} = (1-\cos(y(1-y)))/y$$

So

$$\int_0^1 \int_0^{1-y} \sin(xy)\,dx\,dy = \int_0^1 (1-\cos(y(1-y)))/y\,dy$$

So you could type in:

```
>> f = sin(x*y)
>> int(f,'x')
>> vectorize(subs(ans,{x},{1-y})-subs(ans,{x},0))
>> g = @(y) -1./y.*cos((1-y).*y)+1./y
>> quad(g,0,1)
```

The first three lines do the inner integration and vectorize the result. The fourth line was obtained by pasting the result of the third line after the `@(y)`. When computing `quad`, Matlab might complain a bit when it evaluates g at 0 since it is dividing 0/0 although in fact the limit of this function at 0 is 0. If you can do it, this method is faster and more accurate than the other.

Now let us do the second method. The key fact is that Matlab gives a true expression the value 1 and gives a false expression the value 0. So multiplying the integrand by `x+y<=1` is the same as setting the integrand to 0 outside the region where $x+y \leq 1$.

```
>> ff = @(x,y) sin(x.*y).*(x+y<=1)
>> dblquad(ff,0,1,0,1)
```

Triple Integrals

Triple integrals can be done similarly. Newer versions of Matlab have a triple integrator `triplequad` which integrates over a rectangular parallelepiped. So the best method is to see first if you can evaluate the inner integral, perhaps after changing the order of integration. If you can, you have reduced to a double integral and can use the methods of the previous section. If you can't then you can use `triplequad` to evaluate your integral, (which may take a while). As an example, let us evaluate the integral

$$\int_0^2 \int_0^{(6-3x)/2} \int_0^{6-3x-2y} \sin(x^2 e^y + z)\,dz\,dy\,dx$$

Note that the region of integration is the region in the first octant below the plane $3x + 2y + z = 6$ so it is defined by the inequality $6 - 3x - 2y - z \geq 0$. Likewise, it's projection to the xy plane is the region in the first quadrant satisfying the inequality $6 - 3x - 2y \geq 0$. So for the first method we could do:

```
>> f = sin(x^2*exp(y)+z)
>> int(f,'z')
>> vectorize(subs(ans,z,6-3*x-2*y)-subs(ans,z,0))
>> g = @(x,y) (-cos(x.^2.*exp(y)+6-3.*x-2.*y)+cos(x.^2.*exp(y))).*(6-3.*x-2.*y>=0)
>> dblquad(g,0,2,0,3)
```

But suppose we couldn't do the inner integration, or were just lazy and were willing to let the computer spend its time working out the triple integral. Then we could do:

```
>> ff = @(x,y,z) sin(x.^2.*exp(y)+z) .* (6-3*x-2*y-z>=0)
>> triplequad(ff,0,2,0,3,0,6)
```

Sometimes triple integrals can take a long time to calculate. You can make run faster by doing

```
>> triplequad(ff,0,2,0,3,0,6,.0002).
```

The last number `.0002` gives a tolerance larger than the default tolerance `.000001` so the answer is less accurate but is computed more quickly.

If your region is defined by several inequalities you can just multiply them together. For example, to find the integral of x over the region inside the upper nappe of the cone $z^2 = x^2 + y^2$, inside the cylinder $z^2 + y^2 = 4$ and below the plane $x + 2y + 3z = 5$ you could type:

```
>> ff = @(x,y,z) x .*(z >= sqrt(x.^2+y.^2)) .* (z.^2+y^2 <= 4) .* (x+2*y+3*z <= 5)
>> triplequad(ff,-2,2,-2,2,0,2)
```

I chose my limits as ± 2 since:
a) From $z^2 + y^2 \leq 4$ we know that $\mid z \mid \leq 2$ and $\mid y \mid \leq 2$.
b) Then from $z \geq \sqrt{x^2 + y^2}$ we know $z \geq \mid x \mid$ so $2 \geq z \geq \mid x \mid$.

Matlab 3 assignment: This assignment is due Thursday, Nov. 30. You must provide me with

printed copies of your output with your answers clearly indicated. Also show your work if you do the inner integral calculation by hand rather than using Matlab. As usual, I strongly encourage you to work in groups of two or three, but no more. Please hand in only one assignment per group. As usual you should print all Matlab commands necessary to obtain your answer and should explain your answers adequately.

Problem 1: Find the centroid of the region R which is inside the ellipse $x^2 + 3y^2 = 7$, outside the circle of radius 1 about $(-1, 0)$, and outside the circle of radius 2 about $(3, 1)$. Is the region R simple?

Problem 2: Let R be the region above $z = x^2 + 2y^2 - 3$ and below $z = 5 - x^2 - y^2$. Calculate $\int\int\int_R z/(1 + x^2 + y^2)\, dV$ in two ways, by doing an inner integration and by using `triplequad`. Use a tolerance of `.0002` for triplequad unless you don't mind it taking a very long time.