Homework 1 – due 09/12/03

Math 340

These problems are based on our review of 1-variable calculus.

Problems to be handed in:

1. Write down a function with domain $[0, 1]$ which is not bounded.

2. Use the Mean Value Theorem to PROVE the following fact: If $f : [a, b] \to \mathbb{R}$ is continuous, and differentiable on the set (a, b), and the derivative of f is identically zero on (a, b), then f is a constant function.

3. Suppose $f : (a, b) \to (c, d)$ is a function. Prove that f has an inverse if and only if f is one-to-one and onto. (There are two things to show here: first, assume that the inverse exists and then use that information to show that the function must be one-to-one and onto. Second, assume the function is one-to-one and onto, and use that information to DEFINE an inverse function for f.)

4. Use the Chain Rule to prove the formula from the Inverse function theorem:

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}.$$

5. Show that if f has an inverse, then that inverse is unique. (Hint: suppose g and g' are each inverse to f. Show that $g = g'$ by considering the composition $g \circ f \circ g'$). This result justifies our referring to THE inverse of f.