

Homework 10 – due 12/05/03

Math 340

Problems for practice (highly recommended, but not to be handed in):

3.6.8.

3.7.1, 3.7.3

Problems to be handed in:

1. Problem 3.7.5

2. Problem 3.7.6.

3. Problem 3.7.10.

4. You have fallen into a deep hole, the sides of which can be described as the graph of the function $f(x, y) = 2x^4 - x^2 + y^2 - 20$.

(a) Suppose you are at the very bottom. What are your coordinates? How deep are you? (Is the “bottom” at a unique point?)

(b) Climbing out from the bottom, is there any “ledge” on which you can rest on your way up?

(c) Your coordinates are now $(1, 1)$. In which direction should you climb to ascend most rapidly? In which direction(s) can you go without changing elevation at all?

5. Let A be an $n \times n$ invertible matrix.

(a) Show that A can be written as a product $A = PU$, where P is orthogonal, and U is an upper triangular matrix whose diagonal entries are all positive. (Apply Gram-Schmidt to the columns of A .)

(b) Show that the decomposition in part (a) is *unique* (i.e., if we also have $A = P'U'$ where P' , and U' have the same properties as P and U above, then necessarily $P = P'$ and $U = U'$). This is called the *polar decomposition* of an invertible matrix.