Homework 11 – due 12/12/03

Math 340

Problems for practice (highly recommended, but not to be handed in):

3.3.1, 3.3.2, 3.3.11
3.4.1, 3.4.3

Problems to be handed in:

1. Problems 3.3.13 and 3.3.14.

2. Problems 3.4.2 (use Proposition 3.4.2) and 3.5.4.

3. Consider the symmetric matrix $A = \begin{bmatrix} 1 & -1 & -1 & -3 \\ -1 & 1 & -3 & -1 \\ -1 & -3 & 1 & -1 \\ -3 & -1 & -1 & 1 \end{bmatrix}$. Find the eigenvalues of A. Then find an orthogonal matrix P such that $P^t AP$ is diagonal.

4. (a) Suppose $f : A \to \mathbb{R}$ is integrable, and let $g = f$ except at finitely many points. Show that g is integrable, and $\int_A g = \int_A f$. (Hint: Show it is enough to assume $g = f$ at all but one point, and then do that case.)

(b) Let $f : [0, 1] \times [0, 1] \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} 0, & x \text{ irrational} \\ 0 & x \text{ rational, } y \text{ irrational} \\ 1/q, & x \text{ rational, } y = p/q \text{ in lowest terms.} \end{cases}$$

Show that f is integrable, and $\int_{[0,1]\times[0,1]} f = 0$. (Hint: use the fact that for each n, there are only finitely many p/q in lowest terms with $q \leq n$.)

5. Using the definitions in terms of upper and lower sums, show that the following integral exists, and compute it:

$$\int_{[0,2]\times[-1.1]} x^2 2^y.$$

Now check your answer is correct by computing the integral using iterated integrals (Fubini's theorem).