Homework 3 – due 09/26/03

Math 340

Problems for practice (highly recommended, but not to be handed in):

1.5.1, 1.5.2, 1.5.4, 1.5.6, 1.5.21.

Problems to be handed in:

1. (a) Show that the function $\dfrac{x^3y}{x^6+y^2}$ approaches 0 as (x,y) approaches $(0,0)$ along every line and every parabola through the origin. Hint: the general parabola or line through the origin is of the form $y = ax^2 + bx$ or $x = ay^2 + by$. (To get a line, take $a = 0$.)

(b) Find $\lim_{(x,y)\to(0,0)} \dfrac{x^3y}{x^6+y^2}$, or prove that it does not exist.

2. Problem 1.5.3.

3. Problem 1.5.7 (a), (b).

4. For each function below, find the set of points in $\mathbb{R}^2$ where the function is continuous. Justify your answer using theorems covered in class.

(a)

$$f(x,y) = \begin{cases} \dfrac{\sin(x+y)}{\sqrt{x^2+y^2}}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

(b)

$$g(x,y) = \begin{cases} \dfrac{\sin(x^2+y^2)}{\sqrt{x^2+y^2}}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

5. Problem 1.5.23 (c), (d).