Homework 4 $\frac{1}{2}$ – do before Exam 1, 10/10/03

Math 340

Problems for practice (highly recommended, but not to be handed in):

1.5.23 (b,e)
1.8.1-1.8.4, 1.8.8, 1.8.12,
1.9.3
Section 1.10: Problems 1.6, 1.7, 1.8, 1.15, 1.17, 1.19, 1.27, 1.32.

More Problems:

1. Let $f(x,y) = x^{x^{x^{y^{y}}}} + (\log x)(\arctan(\arctan(\sin(xy))))$. Find $\frac{\partial f}{\partial y}(1,y)$. Hint: There is an easy way to do this.

2. Find the partial derivatives of the following functions.
(a) f(x, y) = sin(xsiny).
(b) f(x, y, z) = x^{y^z}.
(c) f(x, y) = h(x)g(y) (Express answer in terms of derivatives of h and g.)
(d) f(x + y) = g(x² + y) (Express in terms of derivative of g.)

3. Challenge Problem: (You may turn this in for extra credit before Exam 1.) (a) Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is differentiable. Suppose that f_y is identically zero. Prove that f(x, y) is independent of the second variable. If $f_x = f_y = 0$ everywhere, then show f is a constant.

(b) Suppose $A = \{(x, y) \mid x < 0, \text{ or } x \ge 0 \text{ and } y \ne 0\}$. Suppose that $f : A \to \mathbb{R}$ is differentiable, with $f_x = f_y = 0$ on A. Prove that f is constant. Hint: any two points in A may be connected by a sequence of lines each parallel to one of the axes.