Some Practice Problems

Math 340

These are some practice problems from (mainly) the last material we covered in class. You should also be sure to study the rest of the material we covered this term.

1. Consider the ellipsoid centered at the origin

$$E_{abc} = \{\mathbf{x} = (x, y, z) \in \mathbb{R}^3 \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1\},$$

where a, b, c are positive real numbers. Find the volume of E_{abc}.

2. Similarly, we can define the 4-dimensional ellipsoid E_{abcd}, where $a, \ldots, d$ are positive real numbers. Using the answer to part (a) and Fubini's theorem, find the volume of E_{abcd}.

3. Compute the iterated integral

$$\int_0^2 \int_{\sqrt{x}}^{x} e^{y^3/3 - y^2/2}\, dy dx.$$

4. Find an orthonormal basis of eigenvectors for

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 0 & 0 \\ 3 & 0 & -2 \end{bmatrix}.$$

5. Problem 3.6.1.

6. (a) Let A be a symmetric $n \times n$ matrix. Show that the complex eigenvalues of A are all real.
(b) Assume in addition that A is positive definite (by definition, this means the associated quadratic form Q_A is positive definite: $Q_A(\mathbf{x}) > 0$ if $\mathbf{x} \neq \mathbf{0}$). Show that all the eigenvalues of A are positive. (The converse is also true: if all eigenvalues of A are positive, then A is positive definite).