Homework 1 – due 02/06/04

Math 341

Recommended problems:

Problems to be handed in:

1. Let $h(\rho, \theta, \phi)$ denote the change of variable transformation relating spherical coordinates to the usual x, y, z coordinates. Verify that

$$\left| \text{Det } Dh \begin{pmatrix} \rho \\ \theta \\ \phi \end{pmatrix} \right| = \rho^2 \sin(\phi).$$

2. (a) Find the volume of a right cone having height h and base radius R.
(b) Find the volume of the "top" of the "ice cream cone", that is, the region bounded by the surface of the ball $B(0, R)$, the cone emanating from the origin and deviating from the z-axis by the angle ϕ_0, and the disk cut out by the intersection of those two surfaces. (You may either use cylindrical coordinates, or you can deduce it from (a) and what we did in class.)

3. Consider the linear transformation

$$T(u, v, w) = (u - v, v - w, w + u).$$

(a) Describe the image under T of the unit cube $[0.1] \times [0, 1] \times [0, 1]$ in u, v, w-space. Call this image S, regarded as a region in x, y, z-space.
(b) Evaluate the integral of $f(x, y, z) = xy + z^2$ over S.

4. (a) Compute the integral of $f(x, y, z) = xy(x^2 + y^2 + z^2)^{-1}$ over the region

$$S = \{(x, y, z) \mid 1 \le x^2 + y^2 + z^2 \le 5, \ z \ge 0\}.$$

(b) Compute the integral of $g(x, y, z) = xz(1 + x^2 + y^2)^{-1}$ over the region

$$S = \{(x, y, z) \mid 1 \le x^2 + y^2 \le 3, \ x \ge 0, \ 0 \le z \le 3\}.$$

5. Show that the rational numbers in the interval $[0, 1]$ are of measure zero, but not of content zero.