Homework 10 – due 05/07/04

Math 341

Problems to be handed in:

1. Braun 3.10, # 13, 17.

2. Braun 3.11, # 4, 7, 17.

3. Braun 3.12, # 2, 3, 5, 6, 8-10.

4. *Theorem.* Every $n \times n$ matrix with complex entries can be put into *Jordan normal form.*

More precisely, if $A \in M_n(\mathbb{C})$, then there exists an invertible $n \times n$ complex matrix P such that $P^{-1}AP$ is in *Jordan normal form*, that is, it consists of block matrices down the diagonal, each block (called a *Jordan block*) being of form

$$\begin{pmatrix} \lambda & 1 & \cdots & \cdots & 0 \\ 0 & \lambda & 1 & \cdots & 0 \\ 0 & & \ddots & & \\ 0 & \cdots & \cdots & \lambda & 1 \\ 0 & \cdots & \cdots & \cdots & \lambda \end{pmatrix}.$$

(Aside from the block matrices down the diagonal, all the other entries are zero.) For example, the following matrix is in Jordan normal form:

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

and it has three Jordan blocks in it.

Clearly the entries down the diagonals in the various Jordan blocks in $P^{-1}AP$ are nothing other than the eigenvalues of A.

Suppose that given A, you know a matrix P such that $P^{-1}AP$ is in Jordan normal form. Explain how you can use this to completely solve the differential equation $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$. Hint: Can you compute explicitly $e^{P^{-1}APt}$?